Comparability of the non-linear and linearized stability assessment during railway vehicle design

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This paper compares the linearized and non-linear methods of running stability assessment as they are used in railway engineering. It is demonstrated, that both the results of the linear and the non-linear calculations can exhibit perceptible dispersion at one value of equivalent conicity. The comparisons are carried out on two vehicle types, for different calculation methods and wheel/rail contact conditions. It is shown that applying a minimum damping of 5% and full creep coefficients of Kalker's linear theory, the linear critical speeds are below the non-linear critical speeds, allowing applying of linearized calculations in preliminary investigations in an early project phase.

1. Introduction

Stability assessment of rail vehicles is probably the most diversified type of dynamic analysis in railway vehicle engineering. Various feasible methods exist with which computer analyses of running stability can be carried out. On the one hand these consist of linearized analyses of the eigenvalues, on the other of non-linear simulations which can also be realised with varying types of excitation and also assessed according to differing criteria. The question of the applicability of linearized calculations during the railway vehicle design is very topical, as was confirmed during the discussion at the 18th IAVSD Symposium in Kanagawa 2003 [1]. The non-linear studies of rail vehicle stability presented in some publications usually state that the linearized calculation leads to a higher critical speed than the non-linear analysis, and demonstrate this conclusion by a typical form of the bifurcation diagram of a railway vehicle [2-5], see figure 1. It is important to state that the linearized analyses applied during railway vehicle design, as discussed in this paper, do not accord with the calculation of critical speed v_{lin} in figure 1, as they do not apply linearization of wheel/rail contact at the initial wheelset position. Instead, a method of quasi-linearization [6] is applied, with which the linearized wheel/rail parameters are calculated for the specified amplitude of the wheelset lateral movement. The quasi-linearization is a standard method implemented in simulation tools used during railway vehicle engineering.

In this paper the term 'linearized calculation' always signifies a calculation with the application of quasilinear wheel/rail contact model.



Figure 1. Bifurcation diagram with comparison of linear and non-linear critical speed.

The linearization of the contact geometry wheelset/track and the equivalent conicity parameter is not only used for linear calculations, but also to characterise the contact geometry of the pairing wheelset/track combining theoretical or measured wheel and rail profiles. Furthermore, the equivalent conicity is used to characterise the track (combining the measured rail profiles with theoretical wheel profiles), or the geometry of worn wheel profiles (combining the measured wheel profiles with theoretical rail profiles).

The contact geometry applied during stability assessment should cover all possible conditions which can occur during the whole service life of a vehicle, from new to worn wheel and rail profiles, including the whole range of tolerances of wheel and track parameters. A contact geometry wheelset/track, which can be specified as "worst case" from a stability stand point, is typically not known during the engineering. The equivalent conicity is often only one parameter known to characterise the contact geometry, and the upper limit of the equivalent conicity for a wheelset amplitude of 3 mm is applied to assess the bogie stability. This paper concentrates on such a stability assessment at high equivalent conicity. However, the equivalent conicity is applied as input parameter only in the linearized calculations. The conicity values mentioned in relation to the non-linear calculations are for information only and are not applied in the non-linear analyses.

In order to compare the resultant critical speed applying linearized and non-linear calculations it must be considered which parameters and criteria of both methods are to be applied. The paper compares the linearized and non-linear calculations methods under conditions used during railway vehicle engineering. The comparison is carried out on two different vehicles and various pairings wheelset/track representing high equivalent conicity. Firstly, the calculation methods and input parameters applied in linearized and non-linear stability calculations are described. Then a comparison is presented for two different vehicles and various pairings wheelset/track representing high conicity. The comparison is followed by conclusions.

2. Linear calculations with quasi-linear contact geometry wheelset/track

The parameters considered in linear stability calculations are:

- reduction factor of creep coefficients
- linearized parameters of contact geometry wheelset/track
- minimum damping to ensure stable running.

As bogic stability is mainly critical when running on dry and clean rails, the investigations concentrate on dry conditions. In the linear calculations, the creep forces are characterised by the coefficients of Kalker's linear theory [7]. To represent dry and clean rails, full coefficients of Kalker's linear theory are usually applied. In some cases coefficients reduced by a Kalker's factor of 0.67 are recommended [8] for real, slightly contaminated rails. Both factors mentioned are applied in our comparisons, see figure 2.



Figure 2. Comparison of creep force law parameters applied in linear (L1, L2) and non-linear (NL1, NL2) calculations.

The contact geometry wheelset/track is described in linear calculations by equivalent conicity, which is a function of lateral wheelset displacement. The value of equivalent conicity for a wheelset lateral amplitude of 3 mm is used to characterise the contact geometry wheelset/track in railway practice [9, 10].

A specified value of equivalent conicity can represent various combinations of wheelset and track parameters. The equivalent conicity, which is assumed sufficient for linearized calculations, is therefore an insufficient input for non-linear analysis. In addition to the equivalent conicity λ , the linearized wheelset/track model depends upon the contact angle parameter ε and the roll parameter σ [6]. The parameters of a quasi-linear contact model can be determined by

- linearization of non-linear profiles for specified linearization amplitude
- variation of equivalent conicity, setting the other parameters ε and σ as its function.

Applying the quasi-linearization on the effective wheelset/track pairings, the contact parameters often deviate from the parameters specified as a function of conicity. In figure 3 examples of linearized wheelset/track parameters are given which are calculated applying the quasi-linearization for a wheelset amplitude of 3 mm on combinations of typical new wheel and rail profiles and examples of worn rail profiles from straight track analysed for different gauge values. Although the number of investigated pairings wheelset/track is limited, the comparison with the conicity functions applied in simulation tools partially demonstrate large differences of linearized wheelset/track parameters at the same equivalent conicity.

Based on this the results of the linear analyses can therefore deviate amongst each other even at the same value of the equivalent conicity, as demonstrated in an exemplary manner for the conicity 0.4 and 0.6 in figure 4. In particular, the roll parameter varies strongly as well as having a significant influence on the critical speed. For example, at a conicity of 0.4 the roll parameter can be located between 0.04 and 0.12, see figure 3. Consequently, the critical speed of the examined vehicle demonstrates a distribution of almost 40 km/h as shown in figure 4. This example illustrates that, even with the same equivalent conicity, the critical speed using linear calculations can achieve differing values according to the linearization parameters.



Figure 3. Contact angle parameter and roll parameter as function of equivalent conicity as applied for linear calculations in simulation tools and calculated applying the quasi-linearization for a wheelset amplitude of 3 mm on different combinations of wheel and rail profiles and gauge values 1430 – 1438 mm.



Figure 4. Influence of contact angle parameter and roll parameter on the critical speed.

The damping reaches zero at stability limit but, as the linearized methods represent a simplification, the uncertainty of the results should be secured by a safety margin. Therefore, a minimum of 5% of critical damping is usually requested to ensure stable running.

3. Non-linear calculations

The conditions influencing the results of the non-linear stability calculations are:

- friction coefficient between wheel and rail
- contact geometry wheelset/track
- method and criteria applied to assess the stability limit.

In order to model the creep forces, the non-linear simplified theory by Kalker (program FASTSIM) [7] was applied. The friction coefficients 0.4 and 0.5 were applied (figure 2) as those values represent the range usually used to simulate dry wheel/rail conditions in vehicle engineering calculations. The influence of a further increase of the friction coefficient above the value of 0.5 on the critical speed is very small as shown in [11].

Several non-linear calculation methods exist which lead to different results in dependence on the kind of stability analysis and criteria applied, see [11, 12].

One possible classification of the stability analysis methods accords to the definition of the stability limit. From a mechanical viewpoint, a system possessing the capability to oscillate can be viewed as stable if the oscillations decrease following discontinuation of the excitation. Should a limit cycle having constant amplitude arise at a particular running speed, this speed is defined as critical speed. However, in railway practice and in the specifications concerning the vehicle acceptance [9, 10] the stability is defined by way of limit values for the measured signals. Should the limit value be exceeded, the running behaviour is described as being unstable.

A further criterion for the classification can be the output value applied for the analysis. It can be

- wheelset displacement (lateral or yaw displacement)
- sum of guiding forces between wheelset and track (sliding rms value over 100 m distance) as specified for normal measuring method according to [9, 10]
- lateral acceleration on the bogie frame (sliding rms value over 100 m distance) as specified for simplified measuring method according to [9, 10].

Definition of bogie instability according to the US standard 49CFR238 [13], §238.427 is based on lateral acceleration measurement similar to the simplified measuring method according to UIC 518, but without consideration of the influence of the bogie mass. Another criterion still used for on-line surveillance is the peak value of lateral acceleration on the bogie frame, as defined in the (now invalid) version of UIC 515 [14]. The limit value is seen to be exceeded when the value 8 m/s² occurs during more than 6 consecutive cycles with a frequency 4-8 Hz.

Another classification criterion is the type of excitation applied. Differentiation can be made between computer simulations

- without excitation, running on ideal track, starting from the limit cycle and reducing the speed until a stable bogie motion is achieved
- with excitation by a singular irregularity, followed by an ideal track (or with short irregularity sequence followed by an ideal track), with or without variation of the excitation amplitude
- with excitation by stochastic (measured) track irregularity applying criteria used during the vehicle acceptance test.

In the non-linear calculation the wheelset/track contact geometry is described with the effective profiles. The fact that an equivalent conicity value can be represented by varying profile pairings leads to deviations in the results. Furthermore, the resultant critical speed is influenced by the method of analysis, the type of excitation and the choice of criterion, see [11, 12]. This leads to an extensive dispersion of the results, as can be seen in an exemplary manner in figure 5 for two contact geometries 04A and 04B which both exhibit the same equivalent conicity of approximately 0.4. The differences between the results of individual non-linear analyses for the same wheel/rail friction coefficient and the same equivalent conicity value are even larger than with the linearized calculations. Besides the method applied, the differences depend mainly on the shape of the contact geometry as demonstrated in [15].





Depending on the contact geometry, the bifurcation diagram can demonstrate a subcritical bifurcation (figure 1), which means that the limit cycles can only possess an amplitude larger than value A, or a supercritical bifurcation, at which limit cycles can occur with any amplitude within the gauge clearance. The greatest deviations of the resultant critical speeds take place in the case of supercritical bifurcation, when a limit cycle with very small amplitude occurs and this is taken into account as unstable running. In other cases the resultant critical speeds achieve similar values for all methods tested.

For the presented comparison of non-linear and linearized calculations, the non-linear wheel/rail contact geometry leading to supercritical bifurcation was applied. Two non-linear stability analysis methods usually used in rail vehicle engineering were applied:

- damping behaviour behind a single lateral excitation
- run on measured track irregularities and analysis of the sum of guiding forces according to [9, 10].

4. Comparison

As demonstrated, both the results of the linear and the non-linear calculations can exhibit perceptible dispersion at one value of the equivalent conicity. Both types of calculation have in common the fact that the results of the calculations may vary significantly if a constant status of the wheel/track geometry and creep forces does not exist; but this practically never occurs in reality. This situation must be considered during vehicle design, with the aid of appropriate security margins.

The non-linear and quasi-linear calculation methods were compared for two vehicle types represented by models constructed in the multi-body simulation tool SIMPACK, see figure 6:

- four-car articulated vehicle with Jakobs' bogies and yaw dampers
- conventional four-axle coach of a commuter train without yaw dampers.



Figure 6. Vehicle models compared in the study: a four-car EMU (left) and a single coach of a commuter train (right).

The non-linearities in the models mainly consider yaw dampers and bump-stops suitable for linearization considering small displacements about the initial conditions. No friction elements and hysteresis were present in the models used.

The stability assessment was compared by application of

- quasi-linear analyses with variation of the Kalker's factor (0.67 and 1.0) and variation of the minimum damping considered when evaluating the critical speed (0% and 5% of critical damping)
- non-linear analyses with variation of the wheel/rail friction coefficient (0.4 and 0.5) and the wheel/rail pairings to set up the specified conicity (on the one hand by altering the track gauge, on the other by rail profile wearing).

The comparisons can be seen in figure 7 for the four-car articulated vehicle with Jakobs' bogies and in figure 8 for the conventional four-axle coach. Comparing average non-linear results with linearized calculations, the best agreement is between the results for the wheel/rail friction coefficient 0.4 in non-linear calculations and damping 0% together with Kalker's factor of 0.67 in linearized calculations (top left-hand diagrams in figures 7 and 8). For the friction coefficient of 0.5 in non-linear calculations and damping 0% together with full Kalker's factor 1.0 in linearized calculations, the results of linearized calculations are the same, or slightly lower than the non-linear results. For minimum damping of 5% and Kalker's factor 1.0 in linearized calculations are always lower than the results from non-linear analyses (bottom diagrams in figures 7 and 8).

The comparisons demonstrated that, under conditions usually applied during railway vehicle engineering (minimum damping 5%, Kalker's factor 1.0), linearized calculations deliver a lower critical speed than the non-linear analyses for all investigated examples and are therefore on the safe side. This is in contradiction to the statement usually presented and shows that, when commenting the relation of linearized and non-linear stability assessment, we must always consider which parameters and criteria of both methods are to be applied.

5. Conclusions

For the application of the linearized stability calculations during vehicle design in the industry conservative parameters are recommended, in order that the results are on the safe side in comparison to the non-linear calculations. It was demonstrated by comparisons that, applying the minimum damping of 5% and Kalker's factor 1.0, the linearized calculation results are below the non-linear critical speeds.



Figure 7. Comparison of non-linear and linearized calculations for a four-car EMU.



Figure 8. Comparison of non-linear and linearized calculations for a single coach of a commuter train.

The linearized calculations are suited for preliminary calculations, particularly when no complete information concerning the actual contact geometry wheelset/track is available in an early project phase. The verification calculations should then be carried out with the aid of more exact non-linear analyses. These investigations and conclusions concern only vehicles without friction elements. In any case, vehicles with friction elements are not suitable for linearization, and stability assessment can only take place by way of non-linear calculations.

References

- [1] Goodall, R.M. and Iwnicki, S.D., 2004, Non-linear dynamic techniques v. equivalent conicity methods for rail vehicle stability assessment. *Vehicle System Dynamics Supplement* **41**, 791-799
- [2] True, H., 1994, Does a critical speed for railroad vehicle exist? *Proceedings of the 1994 ASME/IEEE Joint Railroad Conference*, Chicago, Illinois, March 22-24, 125-131
- [3] True, H., 1999, On the theory of nonlinear dynamics and its application in vehicle systems dynamics. *Vehicle System Dynamics* **31**, 393-421
- [4] Knothe, K. and Böhm, F., 1999, History of stability of railway and road vehicles. *Vehicle System Dynamics* **31**, 283-323
- [5] Schupp, G., 2004, Computational bifurcation analysis of mechanical systems with applications to railway vehicles. *Vehicle System Dynamics Supplement* **41**, 458-467
- [6] Mauer, L., 1991, The modular description of the wheel to rail contact within the linear multibody formalism. In: Kisilowski, J., Knothe, K., (Eds.) Advanced railway vehicle system dynamics, (Warsaw: Wydawnictwa Naukowo-Techniczne), 205-244
- [7] Kalker, J. J., 1990, *Three-dimensional elastic bodies in rolling contact*. (Dordrecht, Netherlands: Kluwer Academic Publishers)
- [8] ORE B 176, 1989, Bogies with steered or steering wheelsets. Report No. 1: Specifications and preliminary studies, Vol. 2, Specification for a bogie with improved curving characteristics. (Utrecht: ORE,)
- [9] UIC Code 518, 2003, *Testing and approval of railway vehicles from the point of view of their dynamic behaviour Safety Track fatigue Ride quality.* 2nd edition, April 2003 (Paris: UIC)
- [10] EN 14 363, 2005, Railway applications Testing for the acceptance of running characteristics of railway vehicles Testing of running behaviour and stationary tests. (Brussels: CEN)
- [11] Polach, O., 2005, On nonlinear methods of bogie stability assessment using computer simulations. Proceedings of the Institution of Mechanical Engineers, Part F, Journal of Rail and Rapid Transit, 220, No. F1/2006, 13-27
- [12] Polach, O., Vetter, A., 2004, Methods for running stability prediction and their sensitivity to wheel/rail contact geometry. *Extended abstracts of the 6th International Conference on Railway Bogies and Running Gears*, Budapest, 13-16 Sept., 2004, 62-64
- [13] 49CFR238, 2003, FRA Regulations, Title 49: Transportation, Part 238 Passenger Equipment Safety Standards, Revised October 1, 2003. Federal Railroad Administration. Available online at: www.gpoaccess.gov/cfr
- [14] UIC Kodex 515, 1984, Reisezugwagen Laufwerke (1), 2. Ausgabe, 1.1.1984 (Paris: UIC)
- [15] Polach, O., 2005, Influence of wheel/rail contact geometry on the behaviour of a railway vehicle at stability limit. *Proceedings of the ENOC-2005*, Eindhoven University of Technology, The Netherlands, 7-12 August 2005, 2203-2210