On non-linear methods of bogie stability assessment using computer simulations

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Abstract: Stability assessment of rail vehicles is probably the most widespread form of dynamic analysis in railway vehicle engineering. The computer simulations using fully non-linear three-dimensional vehicle models constructed in a modern multi-body simulation tool allow detailed non-linear stability analysis for the specified conditions. However, high sensitivity to the wheel/rail contact conditions and different definitions of stability in mechanics and in railway practise can lead to significant differences between prediction and the measurement. Different methods of non-linear stability analysis, which may be used in industrial applications, are introduced and compared on selected examples of contact geometry wheel set/track with high equivalent conicity. The comparisons show that the linearization of the contact geometry wheel set/track can enable a better assessment of the non-linear stability analyses. A decreasing equivalent conicity function in the range of amplitudes below ~3 mm leads to supercritical Hopf bifurcation with small limit cycles and consequently to largest differences between the methods compared. When excluding the contact geometries leading to supercritical Hopf bifurcation, the results achieved are closer each other, but still with differences in the range of up to 10 per cent. This uncertainty in the stability prediction caused by the method applied must be taken into consideration, in addition to other uncertainties related to vehicle parameters, modelling, etc.

Keywords: rail vehicle dynamics, instability, unstable running, wheel/rail contact

1 INTRODUCTION

The self-excited vibration of wheel sets, running gears, or the whole vehicle constitutes a phenomenon which must be considered during the design of railway vehicles. In particular, so-called bogie instability has safety relevance and should be avoided under all circumstances. With this in mind, stability assessment plays a significant role during railway vehicle design.

The occurrence of wave-like wheel set motion that could lead to bogie instability is a phenomenon already discovered at a very early date of the railway history. However, the necessity of stability investigations was only slowly recognized during the mid-20th century. During 1960–70, a theoretical comprehension of railway vehicle stability emerged as a result of studies founded on linearized models. Representative for this development is the work of Matsudaira, Wickens, De Pater, Joly; see also the monographs of Wickens [**1**], Knothe and Stichel [**2**], and paper by Knothe and Böhm [**3**] for historical overview and further references. At a later date, the non-linearities of wheel/rail combination were also taken into consideration. Regarding the development of the non-linear methods, Moelle and Gasch [**4**] and True [**5**, **6**] can be certainly mentioned if only in a representative manner. True investigated various aspects of the non-linear railway vehicle stability analysis, under application of the continuation-based bifurcation analysis (path-following method). This method has been recently applied by Schupp [**7**] linked up with a multi-body simulation package. Kaiser and Popp [**8**, **9**] have investigated the influence of wheel set elasticity on running behaviour and stability. True and Trzepacz [**10**] have examined the influence of dry friction in the suspension on the stability characteristics of a railway freight wagon wheel set; Stichel [**11**] has investigated the running behaviour of two-axle freight wagons running at stability limit. Zboinski and Dusza [**12**] have examined stability on curved tracks whereby, in this case, the amplitudes of the limit cycles are illustrated as a function of the input parameter without representation of the critical speed.

Today's computer-aided vehicle dynamic analyses enable extensive virtual development of the railway vehicles. However, the stability analysis constitutes the most diversified application of vehicle dynamics. Methods such as linearized and non-linear calculations can be applied in various versions. The methods may vary depending on whether they are based on the theory of mechanics or on experience gained from measurements. The number of options is also widened by the diversity of input parameters and conditions available, making it difficult to compare and analyse both the procedure method and the results of a stability calculation. One reason for the stability analyses diversification stems from the definition of the stability limit and the critical speed. From the standpoint of mechanics, a system possessing oscillation capabilities is viewed as being unstable if it is unable to regain its momentary state and incrementally distances itself from same. When applied to railway vehicles, the terminology 'unstable run' is incorrect from a physical standpoint, as already stated in 1976 by Zottmann [**13**] as well as in several papers by True [**6**]. When a railway vehicle reaches the stability limit, a limit cycle occurs that constitutes a stable status characterized by a higher amplitude level. However, in the case of railway vehicles, the terminology 'Instability' has become customary as well as being applied in specifications [**14**, **15**], and it is for this reason that the wording is applied in this article also. The lowest running speed at which a limit cycle with constant amplitude occurs is referred to as the critical speed. Contrary to theory, in railway practise and vehicle acceptance regulations [**14**, **15**], the stability of bogies or running gears is defined by way of limits applied on the measuring values. If the limit value exceeds, the running behaviour can be described as being unstable. When a limit cycle of the complete vehicle occurs, during which the wheel sets, bogie, and the car body simultaneously oscillate with a low frequency, the limit value of the bogie instability will often not be achieved, and according to those criteria, this behaviour will not be classified as unstable.

The second reason for the large variety of stability analyses is the decisive role played by the wheel and rail contact conditions. Besides the wheel/rail friction coefficient, the geometry of the contact wheel/rail or wheel set/track also plays a decisive role. In railway applications, linearization is largely resorted in order to characterize the contact geometry wheel set/track with only one parameter – the equivalent conicity. The characteristics of the wheel set/track pairing are 'replaced' with an 'equivalent wheel set' with conical wheel tread surface, whereby this 'replacement' only possesses validity for one value of the wheel set lateral amplitude. There exist several methods for determining the equivalent conicity (with the consequence that more conicity definitions become available [**16**]), which may partially lead to differing results for the same conditions, as reported by Bonadero [**17**].

However, even when the focus is on one definition of the equivalent conicity, the interpretation of the commonly held concept of equivalent conicity is still not explicit.

The equivalent conicity of wheel set/track is influenced both at the track side by rail profile, rail inclination, and track gauge and at the wheel set by wheel profile, inside gauge (distance between the wheel's inside faces), and wheel diameter difference between the left and right wheels.

If a conicity value is assessed by one method or another, the same value could be the result of various actual conditions, so that the realization of a predetermined conicity with the non-linear wheel set/track contact is ambiguous.

The specification of the most unfavourable conditions concerning contact geometry wheel/track represents a difficult factor of stability assessment. Both profiles and track gauge may demonstrate large deviations from the nominal parameters. In publications concerning non-linear analyses, calculations are usually presented with new wheel and rail profiles [**4**, **7–10**, **12**, **18**]. However, these do not necessarily lead to the most critical conditions concerning bogie stability (i.e. too high equivalent conicity). The choice of the most unfavourable, yet at the same time a representative contact geometry wheel set/track constitutes an important and until now a seldom investigated task. It can be the case of worn wheel profiles or the case of worn rails with flattening of rail crown occurring on straight track, as presented by Müller [**19**].

The objective of this article is to compare the methods of the non-linear stability analyses from a standpoint of their application during railway vehicle engineering. Computer simulations using fully non-linear three-dimensional vehicle models constructed in the multi-body simulation tool SIMPACK are applied. The most frequently used methods will be compared by application of the contact geometries between wheel set and track, which lead to a high equivalent conicity. However, only the instability of the bogie or running gear will be thereby considered. The low frequency limit cycle vibration of the complete vehicle ('body hunting'), which influences rather the limits of running characteristics than those of the instability, does not stand at the forefront of this investigation. The contact geometry wheel set/track will be realized both by theoretical and measured worn profiles. The details concerning the equivalent conicity only serve as an indication and will not be applied in the calculations.

2 NONLINEAR METHODS FOR THE BOGIE STABILITY ASSESSMENT

There are several possible criteria for the classification of the non-linear method for the bogie stability assessment.

One possible classification is according to the analysed values. It can be as follows:

(a) wheel set displacement (lateral or yaw displacements);
(b) forces between wheel set and track (sum of guiding forces, called also track shifting force, as specified for normal measuring method according to UIC 518 [**14**]);
(c) lateral acceleration on the bogie frame (as specified for simplified measuring method according to UIC 518 [**14**]).

The measurement of acceleration on the car body, as also specified in UIC 518, will not be considered in the comparison of calculation methods. The signals gained on the bogie itself are more suitable for the bogie stability assessment. As all the signals are available in computer simulations, no advantage is gained by applying the car body acceleration for the bogie stability assessment.

A further criterion for the classification can be the definition of the stability limit. From a mechanical viewpoint, a system possessing the capability to oscillate can be viewed as stable if the oscillations decrease following discontinuation of the excitation. If a limit cycle having constant amplitude arises at a particular running speed, this speed is defined as a critical speed. However, in railway practise and in the specifications concerning the vehicle acceptance [**14**, **15**], the bogie stability is defined by way of the limit values of the measuring quantities. If the limit value exceeds, the running behaviour is described as being unstable.

Another classification criterion is the type of excitation applied. Differentiation can be made between computer simulations:

(a) without excitation – running on ideal track, starting from the limit cycle, and reducing the speed until a stable bogie motion is achieved;
(b) with excitation by a singular irregularity, followed by an ideal track (or with short irregularity sequence followed by an ideal track), with or without variation of the excitation amplitude;
(c) with excitation by stochastic (measured) track irregularity as used during the acceptance test of the vehicles.

In the on-track tests with real vehicles, it is impossible to realize the first and the second method. They can be tested only on a test rig, whereas the third method described is used during the acceptance on-track test.

3 COMPARISON OF THE METHODS ON SELECTED EXAMPLES OF WHEEL/RAIL CONTACT GEOMETRY

To compare the methods, a model of a four-car articulated vehicle in simulation tool SIMPACK was used. The friction between wheel and rail was set to 0.4 (dry rail). The results are given for the trailing wheel set of the first bogie, at which the stability limits are first reached.

The methods of non-linear stability analysis were compared with the aid of examples of wheel set/track contact geometries with high equivalent conicity for the nominal gauge value of 1435 mm and with differing form of the equivalent conicity as a function of lateral amplitude of the wheel set. Four wheel set/track combinations with new as well as worn profiles of wheels and rails were chosen for comparison. The shape of the profiles and the contact points of the investigated profile pairs during a lateral wheel set movement are presented in Fig. 1. To compare the contact geometry parameters of the profile pairs, equivalent conicity was calculated by the harmonic linearization method [**20**] available in the preprocessor of the simulation tool SIMPACK. Figure 2 presents the conicity functions calculated for rigid contact as usually used in railway-established practise and for quasi-elastic contact as implemented in SIMPACK [**21**]. The quasi-elastic contact demonstrates more realistic contact conditions than the rigid contact and is therefore applied in the presented simulations.

The value of equivalent conicity for a wheel set lateral amplitude of 3 mm as requested in UIC 518 [**14**] was used to characterize the contact geometry wheel set/track. Two of the investigated contact geometries demonstrate the similar equivalent conicity of about 0.4 (04A, 04B) and the other two the equivalent conicity of about 0.6 (06A, 06B). Although the equivalent conicity at an amplitude of 3 mm is the same, the progression of the conicity as a function of the lateral amplitude demonstrates significant differences. Below the amplitude of 3 mm, one of the wheel set/track pairs in each conicity group demonstrates increasing equivalent conicity (contact geometry A), whereas the other demonstrates decreasing equivalent conicity (contact geometry B) (Fig. 2).

In the following, the methods for the non-linear stability analysis are described and the results for

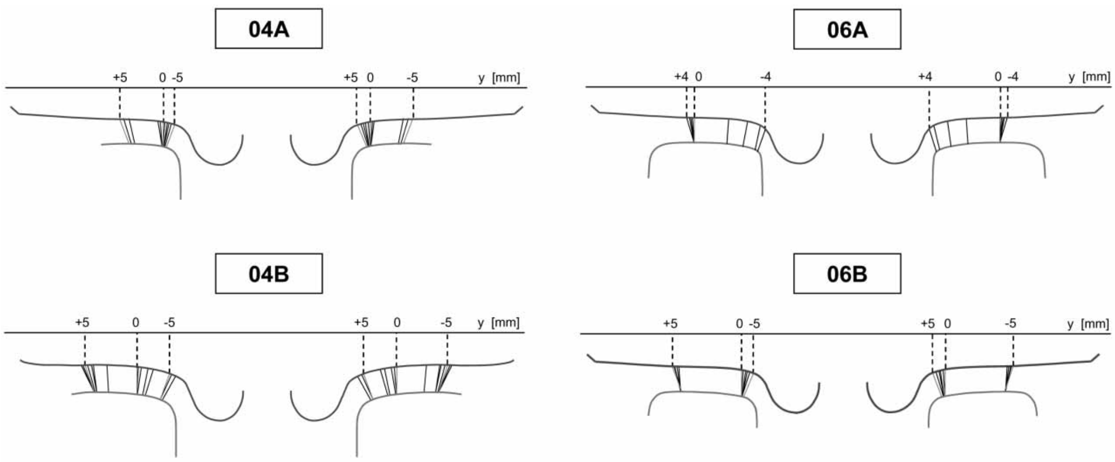

Fig. 1 The contact geometry of the investigated profile pairs

the four variants of the wheel set/track contact geometries mentioned earlier are presented.

3.1 Method without excitation

In this case, a high speed that the bogie moves at in a limit cycle is used as the initial condition, and a continuous speed reduction takes place as is applied, for example, in the investigation concerning the tuning of freight wagon bogies using inter-axle linkages [**22**]. The speed at which the vibrations subside is designated as being the critical speed (Fig. 3). For one contact geometry type (04A, 06A), the vibrations stop abruptly, whereas in the other cases (04B, 06B), the wheel sets continue to vibrate in a small limit cycle, only stabilizing at a significantly lower speed, which subsequently leads to significantly differing critical speeds at the same equivalent conicity.

3.2 Methods with single excitation

By investigating damping behaviour following a single lateral track excitation, stability can be assessed; however, the damping behaviour at the same equivalent conicity can differ for various contact geometries, as can be seen in Fig. 4, for the investigated examples of contact geometries.

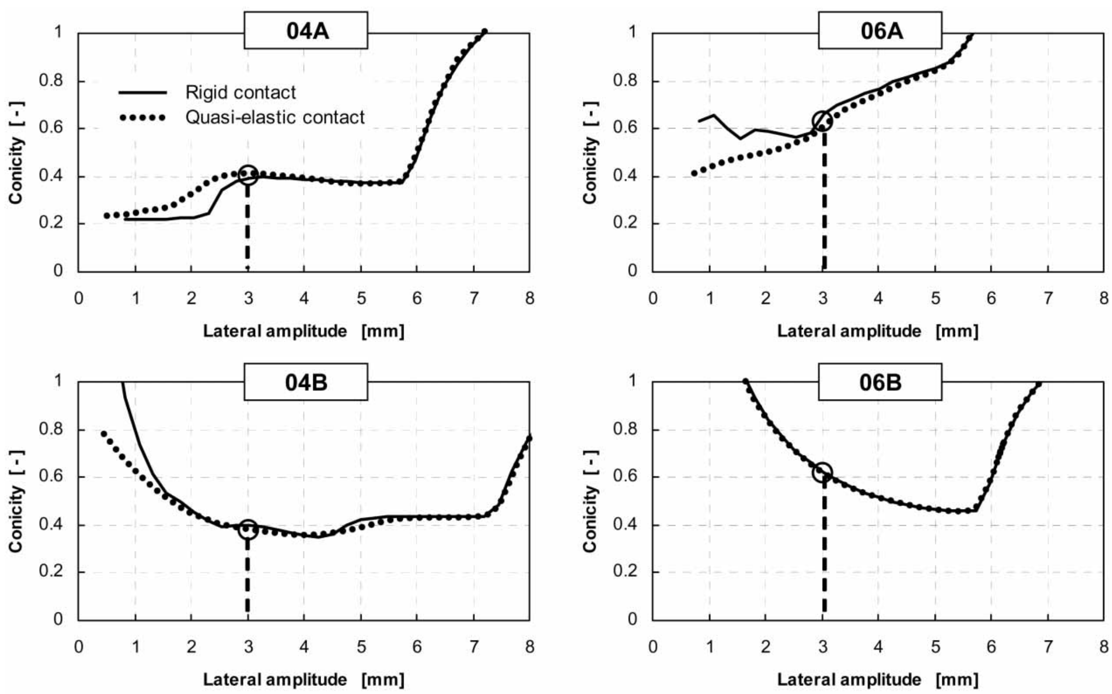

Fig. 2 Equivalent conicity diagrams of examined combinations wheel set/track

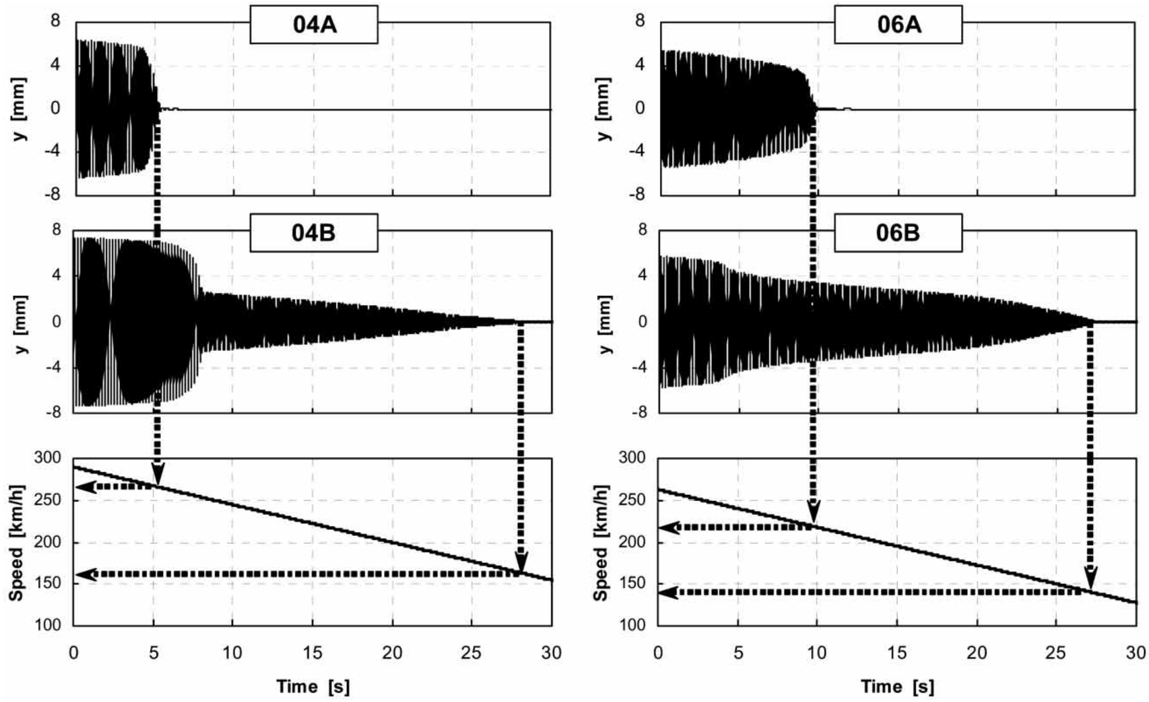

Fig. 3 Simulations of run with decreasing speed

Furthermore, the exciting amplitude also influences the results. If the amplitude of the excitation and the vehicle speed are varied, a diagram of the limit cycle amplitude as a function of speed can be created. For certain range of speeds, the solution can 'jump' between a stable damped movement and a limit cycle depending on the excitation amplitude, as shown in Fig. 5. This behaviour can be explained by the phenomenon called 'bifurcation' which often occurs in non-linear dynamics. The number of solutions can change through bifurcation from an existing solution under a continuous change of the

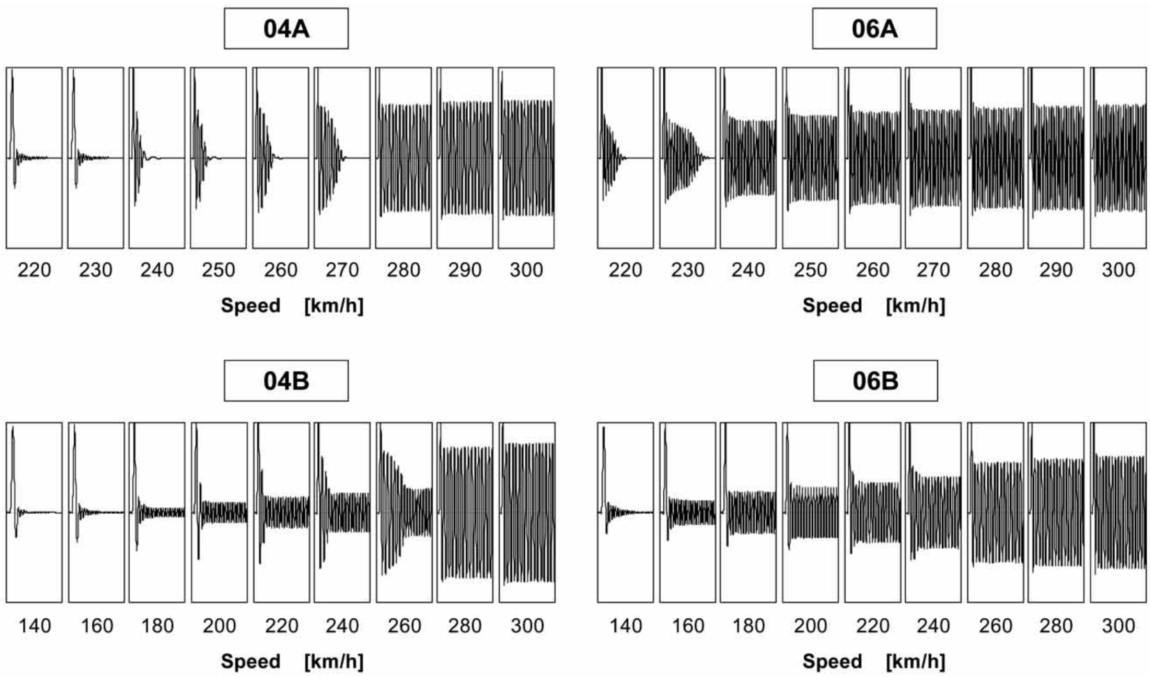

Fig. 4 Simulations of lateral wheel set displacement following a single lateral excitation

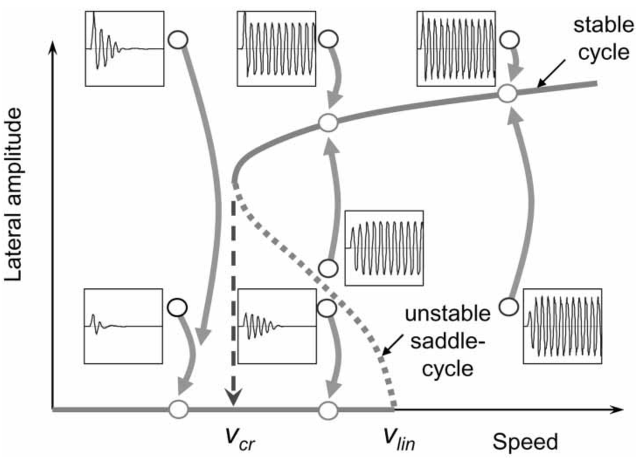

Fig. 5 Bifurcation diagram as a result of the simulations of wheel set behaviour after an excitation

parameter. In this case, the parameter is the vehicle speed and the bifurcation is a Hopf bifurcation, which is a bifurcation of a periodic solution [**6**]. A subcritical Hopf bifurcation exists when the periodic solution is unstable and a supercritical Hopf bifurcation exists when the periodic solution is stable [**6**, **18**] (Fig. 6).

In the case of a subcritical Hopf bifurcation as shown in Fig. 5, the periodic solution is unstable between the speeds v_{cr} and v_{lin}. The non-linear critical speed v_{cr} is defined as speed under which no limit cycle occurs for any exciting amplitude. Above the speed v_{lin}, a limit cycle occurs regardless of excitation amplitude, whereas for the speeds between v_{cr} and v_{lin}, two solutions exist; one stable solution without limit cycle and one with periodic limit cycle.

In accordance with the investigated profile combinations, the bifurcation diagrams assume two basically different forms (Fig. 7). In the first case (profile combinations 04A, 06A), it is a subcritical bifurcation with limit cycle amplitudes >4 mm, whereas in the other case (04B, 06B), the solution corresponds to a supercritical bifurcation diagram, where the amplitude of the limit cycle starts from zero and accumulates with increasing speed.

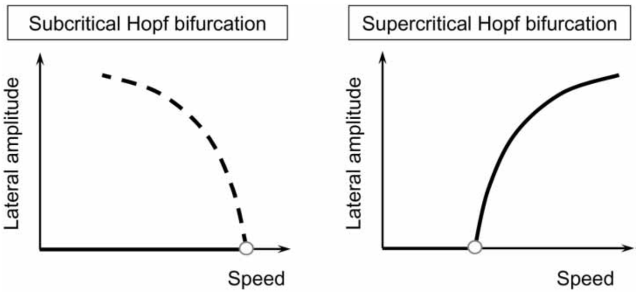

Fig. 6 Hopf bifurcation

3.3 Methods with stochastic excitation and analysis according to criteria for measurements

To assess the bogie stability during the engineering process, the methods specified for measurements and acceptance tests also can be applied. Running on straight track with measured irregularities is simulated and instability criteria for vehicle acceptance tests are applied for assessment.

According to the normal measuring method specified for vehicles having bogies in UIC 518 [**14**] and in the draft of the European standard prEN 14 363 [**15**], the root mean square (r.m.s.) value of the sum of guiding forces Y (track shifting force) is used in full on-track test. The limit value is specified in function of the static vertical wheel load Q_0 in kN as

$$(s\Sigma Y)_{\mathrm{lim}} = \frac{1}{2}\left(10 + \frac{2Q_0}{3}\right)(\mathrm{kN}) \tag{1}$$

which is a half of the track shifting force limit according to Prud'homme. The r.m.s. value is analysed as a continuous average value over 100 m distance calculated with steps of 10 m.

Another practise for proving stability in engineering applications is to measure or to calculate accelerations on the bogie frame. The simplified measuring method, according to references [**14**, **15**], uses acceleration $\ddot{y}^+$ filtered with band-pass filter $f_0 \pm 2$ Hz, where f_0 is the frequency of unstable bogie oscillations. The investigated signal, the r.m.s. value over 100 m distance calculated with steps of 10 m, should be compared with the limit value specified in function of bogie mass m_{b} in tons as

$$(s\ddot{y}^+)_{\mathrm{lim}} = \frac{1}{2}\left(12 - \frac{m_{\mathrm{b}}}{5}\right)(\mathrm{m/s^2}) \tag{2}$$

Definition of bogie instability according to the US standards is based on acceleration measurement similar to the simplified measuring method according to UIC 518, however, without considering the influence of the bogie mass. According to 49CFR238 [**23**, section 238.427], an occurrence of bogie hunting oscillations should be detected by measurement of lateral acceleration on the bogie frame, filtered by a band pass of 0.5–10 Hz. The bogie hunting is defined as an excess of the r.m.s. value of 0.4 g (=3.92 m/s^2) for 2 s. The same limit is also specified in 49CFR213 [**24**, section 213.333], for detection of bogie hunting by automated vehicle inspection systems. The US criteria were not applied in the presented comparisons.

Another criterion still used for on-line surveillance is the peak value of lateral acceleration on the bogie frame, as defined in the (now invalid) version of UIC 515 [**25**]. The limit value is seen to exceed when the

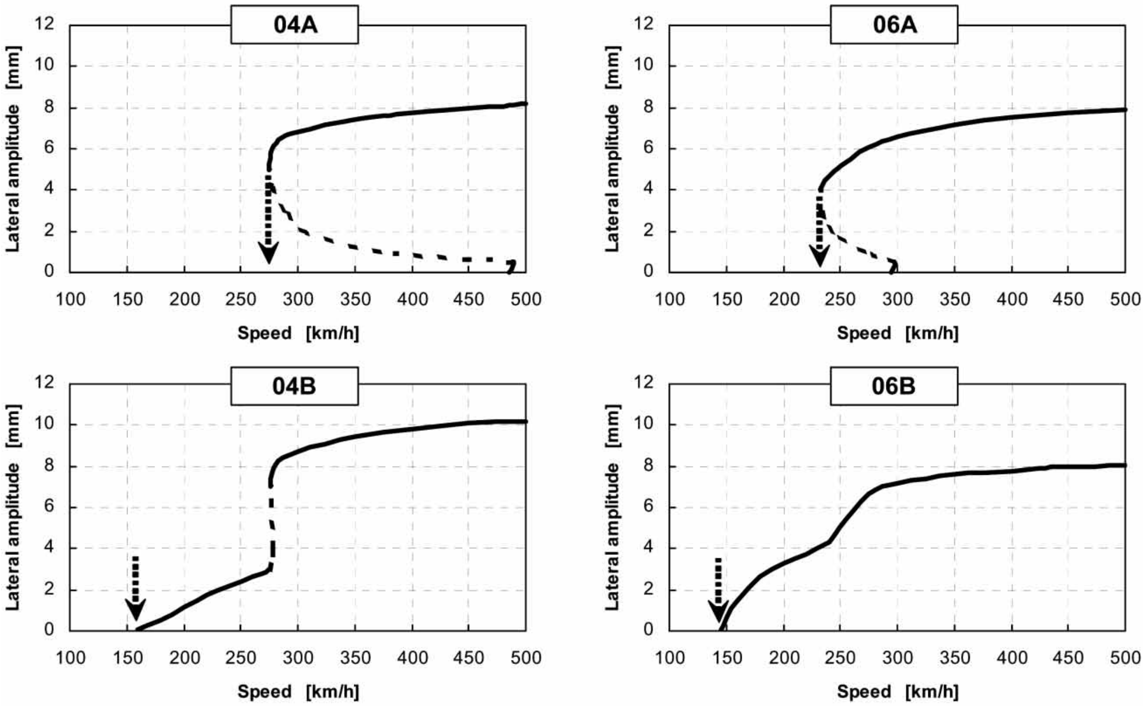

Fig. 7 Bifurcation diagrams for investigated profile combinations

value of 8 m/s^2 occurs during more than six consecutive cycles with a frequency of 4–8 Hz.

The critical speeds evaluated using measuring criteria mentioned earlier were applied in the presented comparison. The speed, at which the bogie instability limit is just achieved, was referenced as critical speed. The evaluation of limit exceedance from the time history plots of the measured values is illustrated exemplarily in Fig. 8 for the wheel/rail combination 04A.

A comparison of the results normalized with the limit value can be seen in Fig. 9. The criteria investigated are comparable in the presented example of the trailing wheel set of leading bogie where the critical speed is first achieved. However the criterion of the r.m.s. value of lateral acceleration at bogie frame leads to a slightly lower permissible speed for the investigated vehicle. In contrast to the method without excitation, the results for both contact geometries for the same value of equivalent conicity are located close to each other in this case.

The situation is completely different in the case of the leading wheel set (Fig. 10) in which leading and trailing wheel sets are compared for the profile combination 04A. The limit value of the acceleration will exceed at 270 km/h, whereas the limit value of the sum of the guiding forces is still not reached, even at 320 km/h. If the maximum values of the complete vehicle are considered, the first exceedance of the limit value takes place at 270 km/h. The criterion of lateral acceleration on the bogie frame delivers comparable results for the measuring value above the leading and trailing wheel set, whereas the criterion of the sum of the guiding forces demonstrates excessive variations between the individual wheel sets. Therefore, must be taken into consideration that for stability assessment (both by applying computer simulation and during the measurement), all wheel sets of the bogie or vehicle must be examined and the highest value compared with the limit value.

The investigated criteria of r.m.s. values presented in Fig. 9 show significantly different progressions from case to case. A sudden stepwise increase as well as a constant growth with different gradients can be observed dependent on the wheel/rail contact geometry. Hence, the stability assessment by analysis of 'reserve' between measured values and stability limit according to measuring criteria [**14**, **15**] does not yield sufficient information to estimate the critical speed. The 'safety margin' to the critical speed cannot be assessed unless the critical speed was identified. This is valid in the same manner for the simulations as well as for measurements. This also explains the difficulty in verifying the simulation results by measurements if the critical speed was not achieved during the tests. In contrast, however, a properly designed vehicle should not reach the stability limit during the test runs.

To investigate the influence of track irregularity, for profile combination 04A, the applied track irregularities were scaled with factors of 0.25, 0.5, and 2.0 and compared with simulation results of previous track

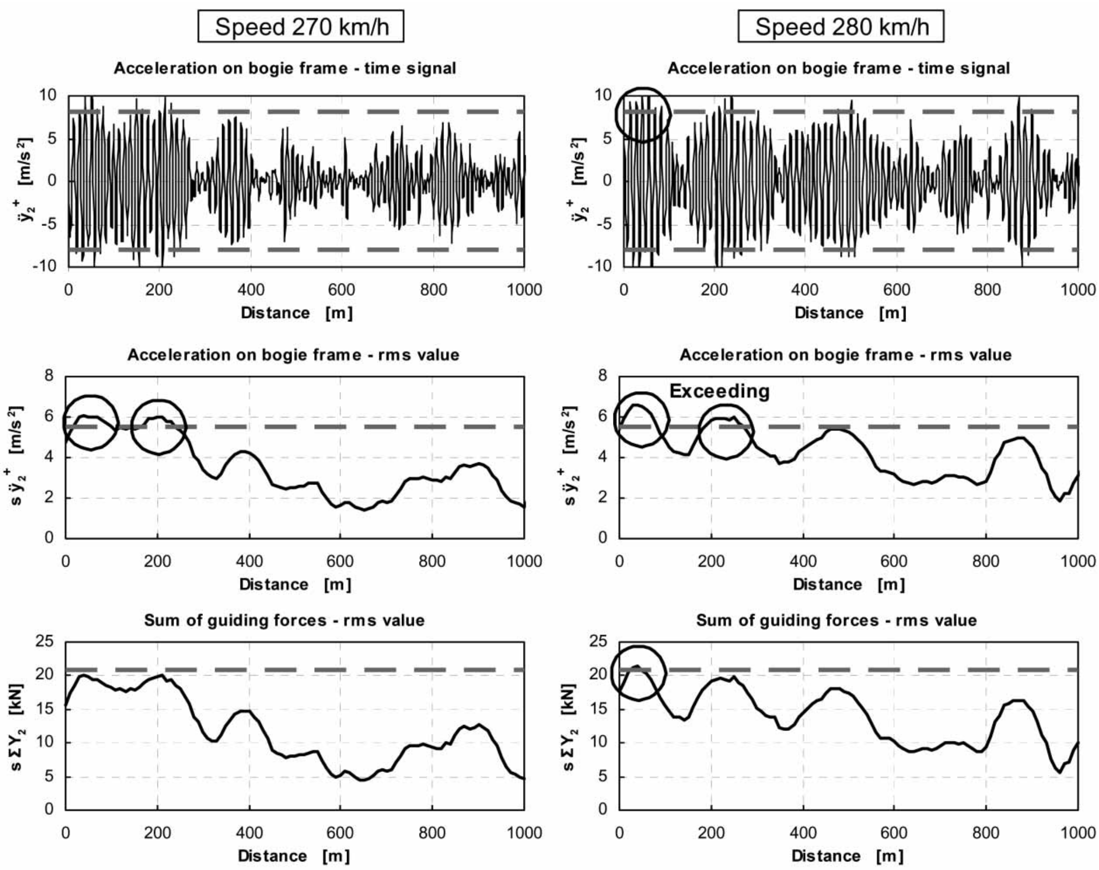

Fig. 8 Stability analysis by simulation of run on track with irregularities (case 04A)

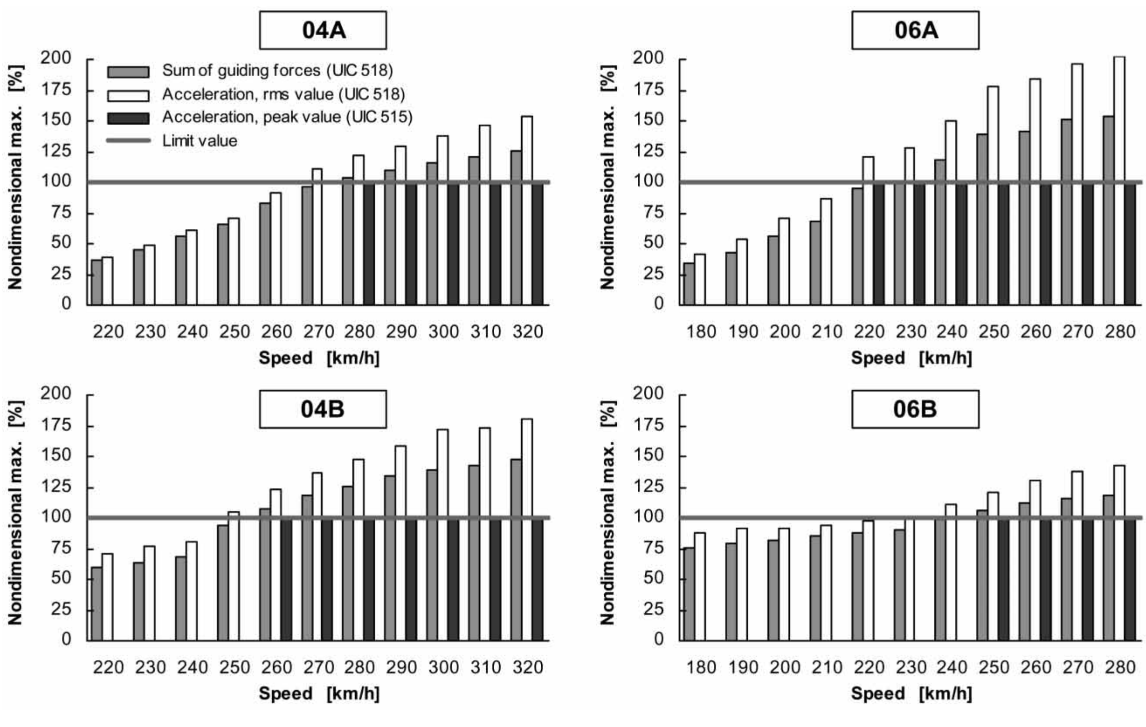

Fig. 9 Normalized results of stability analysis running on track with irregularities. The criterion according to UIC 515 is presented as follows: 0 per cent not exceeded; 100 per cent exceeded

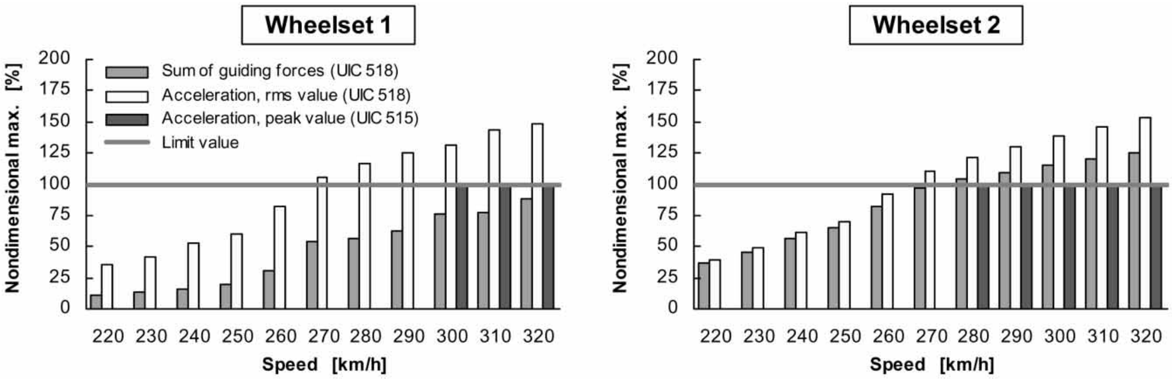

Fig. 10 Comparison of normalized stability analysis results on the leading and trailing wheel set or above it, respectively (case 04A)

irregularities. Figure 11 clearly illustrates that, at increasing amplitude values, the difference between the value of the examined criterion and the limit value decreases. However, the stability limit exceedance only relocates itself slightly. An exception is constituted by the track irregularity with factor 0.25, which demonstrates >50 per cent reserve to limit value at 310 km/h, but alters abruptly at 320 km/h to an exceedance of the limit values. This behaviour can be explained with the aid of the bifurcation diagram (diagram 04A in Fig. 7). For the scale factor 0.25, the highest peak-to-peak lateral excitation which is included in the applied track irregularity achieves 1.9 mm. This value is below the dashed section of the limit cycle curve up to the speed of 313 km/h. The wheel set vibration will therefore decay for speeds <313 km/h as shown in Fig. 11. This example demonstrates that, to decisively investigate the stability limit, the excitation must be sufficiently large.

3.4 Combination of single excitation and criteria from measurements

To interpret the inter-relationships between the measurement limit values, calculations with an excitation by single irregularity with 8 mm amplitude are carried out and the behaviour of the vehicle evaluated according to measurement criteria after the transient has subsided. The calculation procedure is illustrated in Fig. 12. For the speed of 260 km/h, the evaluated criteria are below the limit after the transition, whereas for 280 km/h they are above the limit value. The results for the investigated wheel set/track contact geometries are presented in Fig. 13. If the presented values are greater than zero, this indicates that the wheel set is vibrating with a limit cycle. As can be seen from the results, for one type of the wheel/rail contact geometry, a limit cycle evolves abruptly, leading to exceedance of the limit values for bogie instability, whereas in another case, limit cycles evolve at a significantly lower speed, which however lie beneath the limit values and slowly increase at rising speed. If only the exceedance of the limit values is compared, both the earlier cases will achieve approximately the same critical speed at the same equivalent conicity. However, if the presence of a limit cycle is viewed as constituting the stability limit, the critical speeds will differ significantly even at the same conicity. It can be concluded that although in case of subcritical Hopf bifurcation, the presence of a limit cycle is approximately in line with the measurement

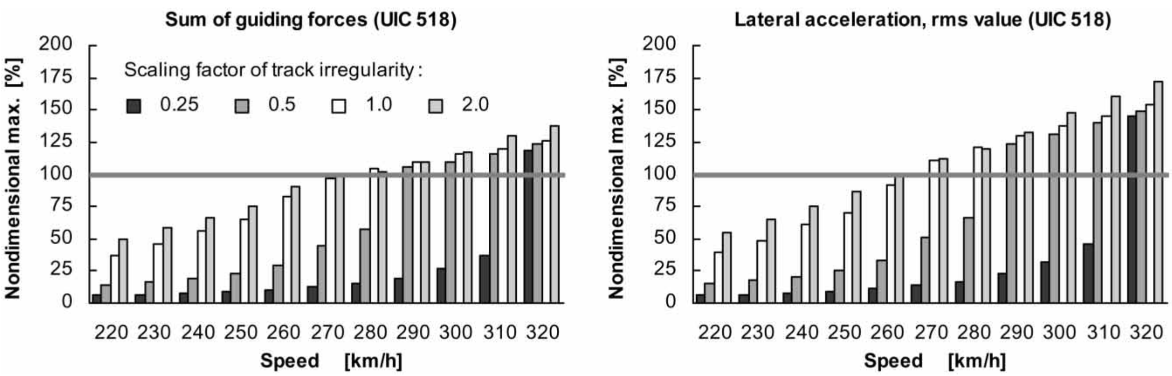

Fig. 11 Influence of track irregularity on the normalized results of the stability analysis (case 04A)

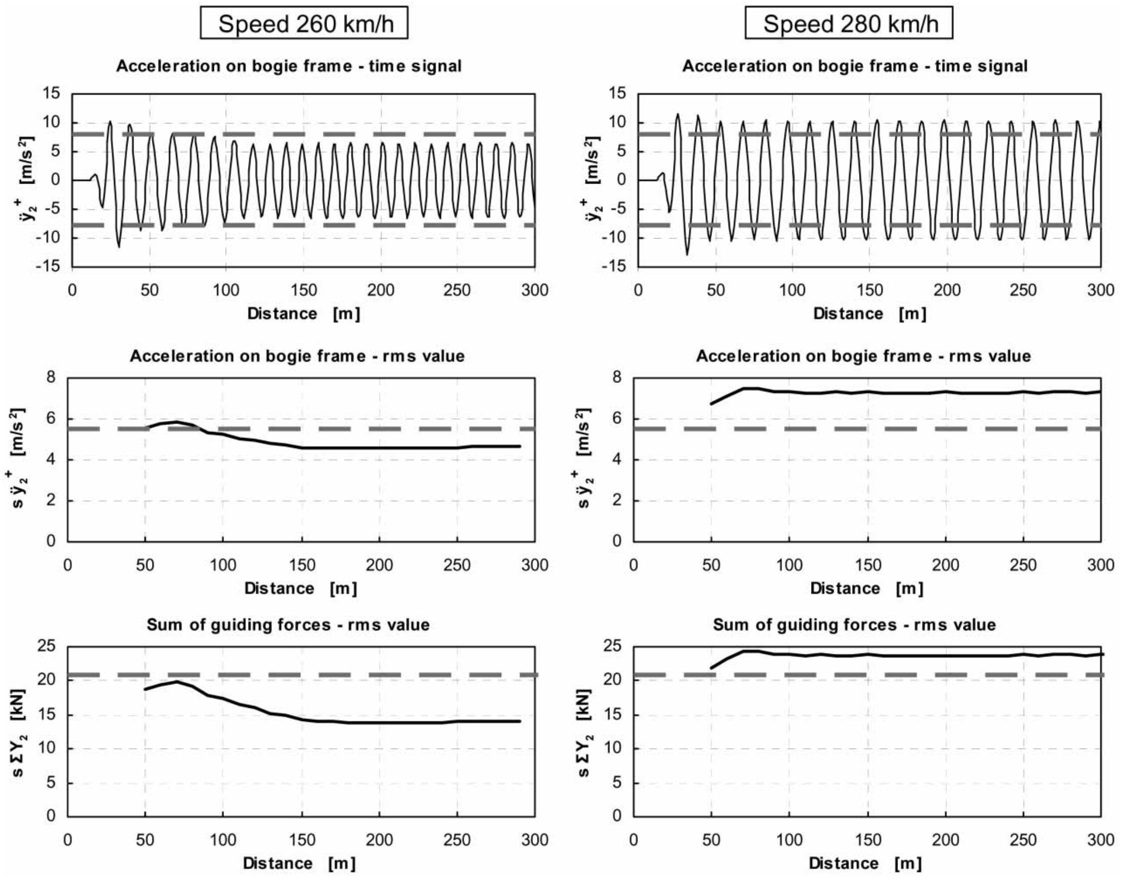

Fig. 12 Simulation of behaviour after single lateral excitation analysed according to the criteria for measurement

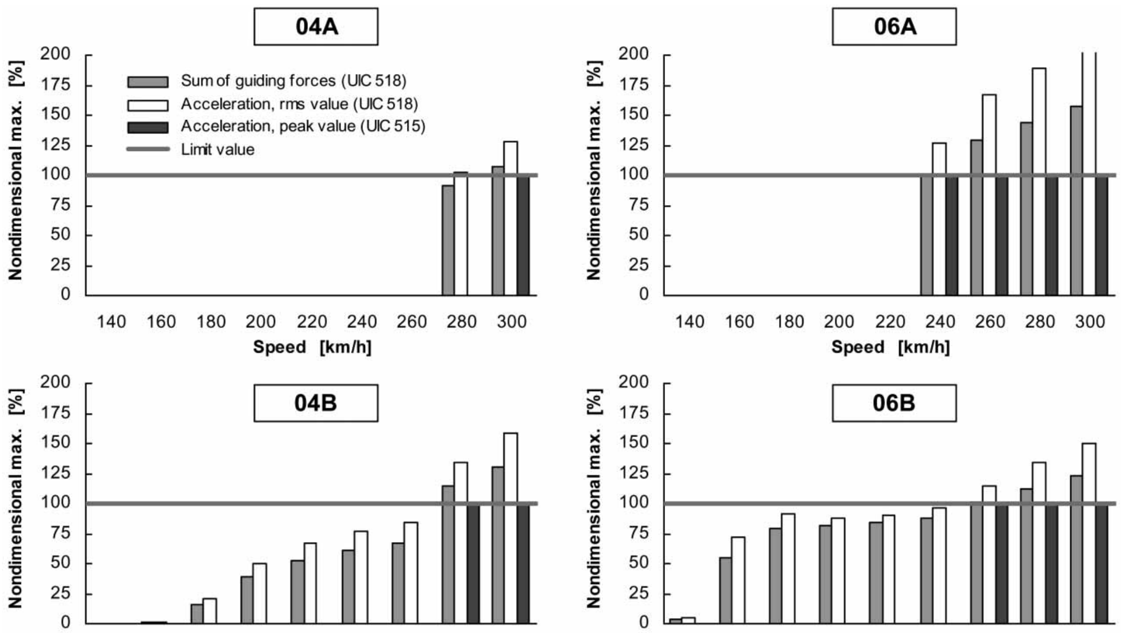

Fig. 13 Normalized results of stability analysis using single lateral excitation and criteria for measurements

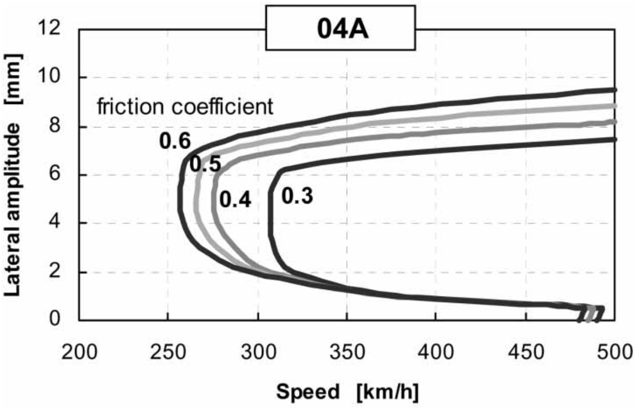

Fig. 14 Influence of wheel/rail friction coefficient on the bifurcation diagram (case 04A)

criteria of bogie instability, in case of supercritical Hopf bifurcation, there is significant difference between an occurrence of a limit cycle and the instability criteria.

3.5 Sensitivity to wheel/rail friction coefficient

As well known, the critical speed decreases at increasing friction coefficient between wheel and rail. Owing to this, the stability assessment should take place under dry wheel/rail contact conditions. During simulation, the value 0.4 is most usually applied for dry conditions in wheel/rail contact, and this value also applies for calculations presented earlier. A value of friction coefficient around 0.5 and, as an exception, even higher values have also been reported. To illustrate the influence of the friction coefficient, values 0.3, 0.4, 0.5, and 0.6 (which can all be considered as dry) have been compared for the profile combination 04A when calculating the bifurcation diagram (Fig. 14) and applying the simulation of run on track with measured irregularities (Fig. 15). From the comparisons, it becomes apparent that the decrease in the critical speed between the friction values 0.3 and 0.4 is high. Between 0.4 and 0.5, the critical speed also decreases lightly, whereas the further decrease in friction coefficient higher than 0.5 is very small or negligible. It is the opinion of the author that a friction value between 0.4 and 0.5 is recommended during stability analysis simulations in order to ensure stable vehicle behaviour under dry conditions.

3.6 Comparison of resultant critical speeds

A comparison of the critical speeds determined by individual non-linear methods is shown in Fig. 16. The lowest critical speeds will be usually achieved in cases without excitation, if the simulation at a high speed commences with a limit cycle and the vehicle is stabilized by a decrease in speed. The critical speeds identified by a single lateral excitation applying the measuring criteria lie up to 20 km/h higher than simulations of running on track with measured irregularities and analysed applying the same measuring criteria. The critical speeds identified by normal measuring method according to UIC 518 (measurement of sum of guiding forces) are the same or higher than the results obtained applying the simplified measuring method according to UIC 518 (measurement of accelerations on the bogie frame), with differences up to 15 km/h.

The greatest deviations from the critical speed take place in case of supercritical bifurcation when small limit cycles occur (04B, 06B), and these are taken into account for the evaluation in conformance with the principles of mechanics.

In other cases, the resultant critical speeds achieve similar values for all methods. However, even in the presented examples of subcritical bifurcations, the differences amount up to 10 per cent at the

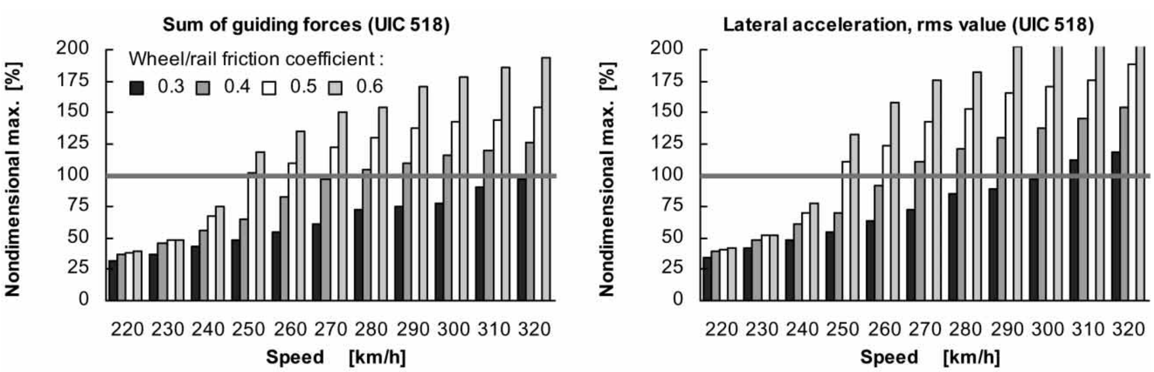

Fig. 15 Influence of wheel/rail friction coefficient on the normalized results of the stability analysis (case 04A)

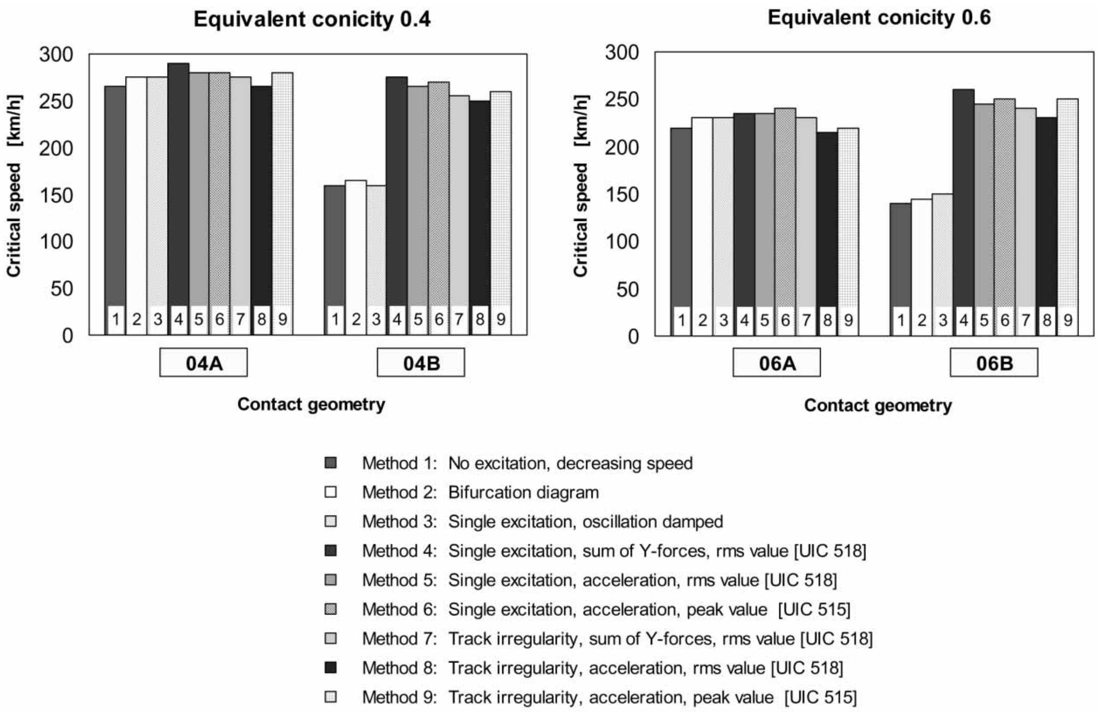

Fig. 16 Comparison of critical speeds calculated applying different methods of non-linear stability analysis

same wheel/rail contact geometry depending on the applied method.

4 DISCUSSION

All the methods proposed can be described as being well suited; however, the properties of the wheel/rail contact geometry have to be taken into consideration. The linearization of the wheel/rail contact geometry can enable a better judgement of the simulation results, if the same contact model (rigid or elastic) is applied for contact geometry analyses as well as for simulations. The gradient of the equivalent conicity in function of wheel set lateral amplitude in the range between 0 and $\sim$3 mm suggests the behaviour to be anticipated at the stability limit. A decreasing equivalent conicity function in the range of amplitudes $<$3 mm leads to low limit cycles (supercritical Hopf bifurcation), whereas the increasing or constant equivalent conicity function is linked with an abrupt transition from stable behaviour to a pronounced limit cycle (subcritical Hopf bifurcation).

The recommendations mentioned were applied when choosing the wheel and rail profiles to represent contact geometries specified by the equivalent conicity value in further comparisons. The required equivalent conicity was achieved, on the one hand, using standardized wheel profile S1002 and rail profile UIC 54E in combination with gauge narrowing and, on the other hand, choosing wheel profile S1002 and measured worn rail profiles in combination with nominal gauge of 1435 mm. For the stability assessment, the following methods were compared.

1. Damping behaviour behind a single lateral excitation.
2. Running on track with measured irregularities and analysis of:
   (a) r.m.s. value of the sum of guiding forces (UIC 518 [**14**], normal measuring method);
   (b) r.m.s. value of the acceleration on the bogie frame (UIC 518 [**14**], simplified measuring method);
   (c) peak value of the acceleration on the bogie frame (UIC 515 [**25**]).

The four methods presented were applied on two vehicle models:

(a) a four-car articulated vehicle with three Jakobs' bogies and two end bogies, with yaw dampers on all bogies (vehicle A);
(b) a single conventional railway coach with two two-axle bogies without yaw dampers (vehicle B).

Two levels of the wheel/rail friction coefficient of 0.4 and 0.5 were used.

As can be seen on the results presented in Fig. 17, the critical speeds identified by different methods are very similar. The tendency of decreasing critical

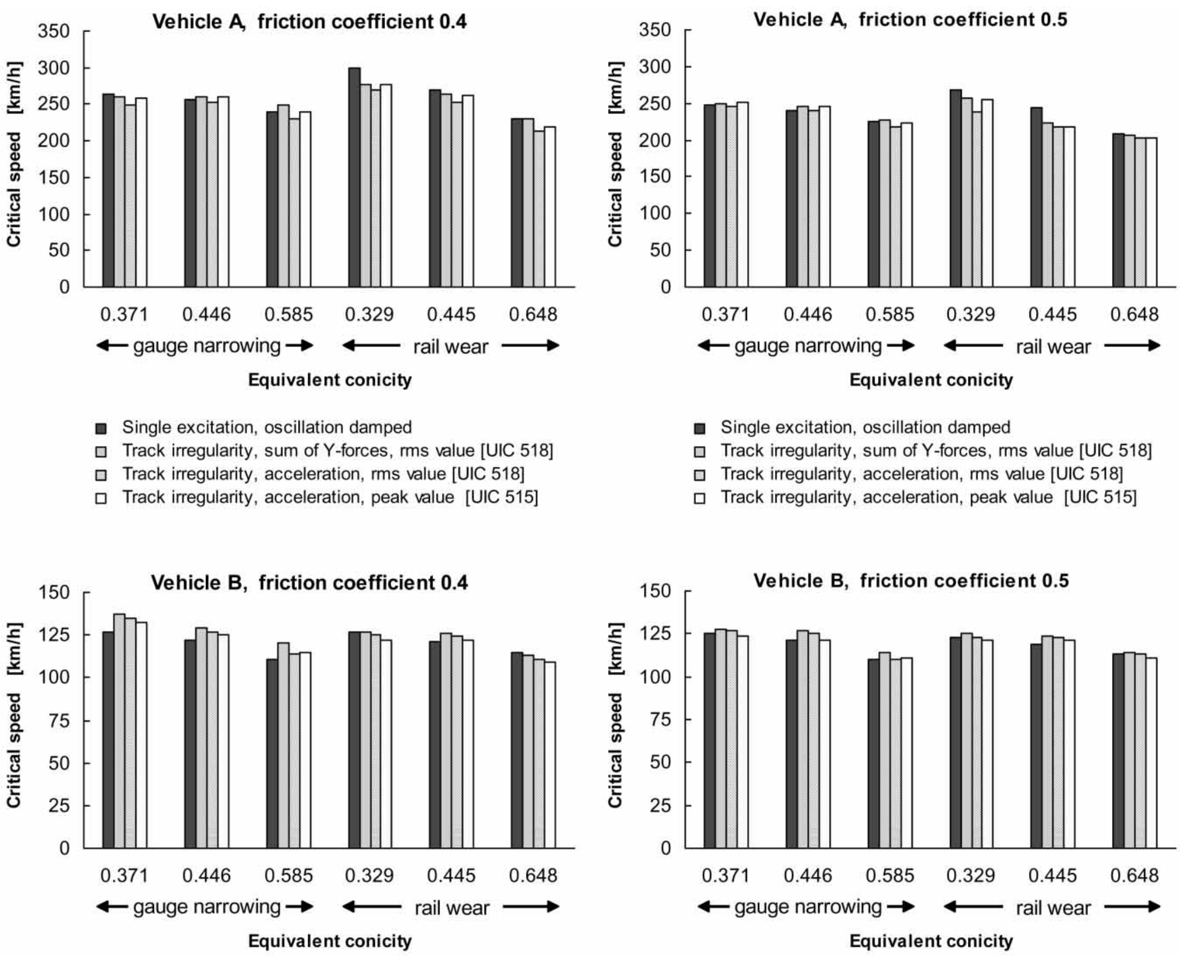

Fig. 17 Comparison of critical speeds calculated for specified equivalent conicity values. The non linear contact geometry wheel set/track was realized by gauge narrowing and by application of worn rail profiles, while respecting the recommendations described

speed with increasing equivalent conicity is also visible; however, the differences between the results for the same equivalent conicity obtained by different methods are sometimes in the same range as differences between the results for differing conicities. The differences between the critical speeds calculated applying different methods for the same wheel rail contact geometry and friction coefficient lie between 2 and 11 per cent for vehicle A and between 3 and 8 per cent for vehicle B. This uncertainty in the stability prediction must be taken into consideration during the design of the vehicle in addition to other uncertainties related to vehicle parameters, modelling, and so on.

The influence of the friction coefficient is interesting. At vehicle A, further decrease in the critical speed can be observed for increased friction coefficient, whereas vehicle B achieves the saturation for the friction coefficient 0.4; there is hardly any decrease in critical speed, increasing the friction to 0.5. Furthermore, different sensitivity to the friction coefficient can be evaluated for different contact geometries of wheel set/track.

The investigation confirms the equivalent conicity to be a useful parameter for the general characterization of the contact wheel set/track. However, it is not a sufficient input parameter for an exact prediction of the critical speed. A more exact stability assessment is only possible specifying detailed description of the contact wheel set/track including rail profiles, rail inclination, track gauge, inside gauge of the wheel set, and difference between left and right wheel diameters.

5 CONCLUSIONS

The stability assessment of a railway vehicle depends significantly – besides other parameters – on the wheel/rail contact and on the method chosen for the investigation. For the analysis of the worst case situation with regard to the bogie stability, a contact geometry wheel set/track with high equivalent conicity, high friction coefficient, and large track irregularities should be applied. The stability assessment can lead to significant differences between prediction and measurements, if the conditions of the contact wheel set/track are not specified precisely enough.

The equivalent conicity provides a useful parameter for the general characterization of the contact wheel set/track from a stability standpoint; however, it is not sufficient input information for an exact prediction of the critical speed. A more exact stability prediction is only possible specifying the wheel/rail contact geometry through the depiction of the wheel and rail profiles, rail inclination, track gauge, inside gauge of the wheel set, and difference between left and right wheel diameters.

The linearization of the wheel/rail contact geometry can enable a better assessment of the non-linear stability analyses. The gradient of the equivalent conicity as a function of wheel set lateral amplitude below $\sim$3 mm suggests the behaviour to be anticipated at the stability limit. A decreasing equivalent conicity function in the range of amplitudes between 0 and 3 mm leads to supercritical bifurcation with low limit cycles, whereas the increasing or constant equivalent conicity function is linked with subcritical bifurcation characterized by an abrupt transition from stable behaviour to a pronounced limit cycle.

The definitions of stability in mechanics and in railway practise are not identical. The individual non-linear methods for stability analysis can therefore lead to differing results. A complete assessment of the behaviour to be anticipated can only be achieved by way of a bifurcation diagram calculation. If a wheel set/track geometry is available leading to a supercritical bifurcation, the differing definitions of stability limit can then lead to larger differences in the results. In this case, limiting cycles evolve with small amplitudes and a significant distance from the flange contact. If the limit cycles with small amplitudes are not taken into consideration, the results of the examined non-linear stability analysis methods show smaller discrepancies.

The presented comparison demonstrates that – in spite of highly sophisticated computer aided stability assessment – differences in the critical speeds up to 10 per cent can occur just caused by the method applied, using the same simulation tool, the same vehicle model, and the same wheel/rail contact conditions. This uncertainty in the stability prediction must be taken into consideration during vehicle design in addition to other uncertainties related to vehicle parameters, modelling, and so on.

The conclusions stated earlier are based on the analysis of modern passenger railway vehicles without friction elements in the suspension, which even today are regularly applied for freight wagons. The discontinuity of the friction element force may lead to different behaviour at the stability limit. At present, this topic is the subject of intensive investigation [**10**, **11**] and requires further thorough research until it will be possible to explain this behaviour.

Only through accurate specification of the wheel/rail conditions, careful choice of method, and sophisticated modelling can a reliable stability prediction and compliance between computer simulation and running behaviour in operation be achieved.

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