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# Simulations of Running Dynamics for Vehicle Acceptance: Application and Validation

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### Abstract

Multi-body simulation tools are used in rolling stock design and development to optimise the vehicle parameters and to conduct a wide range of investigations. Although the vehicle acceptance is traditionally based on physical testing, the use of simulations as a part of vehicle authorisation is increasing. This state of the art paper reviews the recent status of conditions for the application of simulations in the context of vehicle authorisation and summarises the progress of investigations related to the methodology and criteria for a reliable model validation.

**Keywords:** Rail vehicle, running dynamics, simulation, vehicle acceptance, authorisation, validation.

## **1** Introduction

During the decades, multi-body simulations have developed from routines used by researchers and engineers to become mature and reliable programs used in powerful simulation packages. Today nearly every newly developed railway vehicle undergoes a thorough analysis by means of multi-body dynamics simulations.

With the advent of analogue and then digital computers it became possible to solve equations describing the behaviour of the wheelset and rail vehicle for real problems. The early codes tended to split up the types of behaviour to simplify the calculation [1]. Programmes for calculation of steady-state curving equilibrium were developed and linearised analyses applied to predict limits to stable running. As computing power developed, powerful numerical methods were applied in the time domain and three dimensional, fully nonlinear models could be handled. Commercial tools for railway vehicle dynamics have been available from 1980s, alongside with a large number in-house tools.

The results from early simulation codes were compared with measurements to approve that they correspond to the behaviour of the actual vehicles [2], [3]. Although those papers are entitled as validation, they represent rather a verification of the modelling techniques used in the mathematical models and computer codes. The verification of simulation codes and their benchmarking provided the basis for the usage of simulations in the development of new vehicles.

A review of multi-body computer codes for vehicle dynamics and their benchmark on representative problems from rail and road vehicle dynamics was published in 1993 [4]. A comparison of simulation codes related to railway applications represented the benchmark ERRI B176 [5]. Altogether 5 railways companies, 11 rolling stock suppliers and 2 research institutes expressed their interest in participation and received the tasks in April 1991 [6]. The benchmark contained calculations using linear and nonlinear models of a passenger coach. The power spectral density of measured track irregularities provided within this benchmark as well as the linearised description of wheel rail contact geometry have often been referenced in subsequent investigations of running dynamics.

The Manchester benchmarks published in 1999 [7] specified simulations of two vehicles – a passenger coach similar to the ERRI-Benchmark vehicle and a two-axle freight wagon with friction suspension. Four track cases were defined to allow comparison of the capabilities of computer simulation packages. Simulations were carried by the suppliers of the five major commercial simulation tools (VAMPIRE, GENSYS, SIMPACK, ADAMS/Rail-MEDYNA, and NUCARS) and the results presented in [8].

The systematic use of multi-body simulation tools in the rolling stock industry started in 1990s. Advanced industrial application of simulations is described by Ofierzynski [6] in his paper from 1994. The model building (topology, modelling of suspension components, modelling of wheel/rail contact, track model, preparing the model data), plausibility checks, verification techniques as well as processing and organisation (form and development of technical documentation) are explained and examples presented.

The modern multi-body simulations software packages are capable of handling mechanical and multidisciplinary systems of great complexity. They are used by vehicle and component manufacturers, operators, infrastructure companies, consultants and engineering service providers, research institutes and universities. An overview of the software packages used for railway vehicle dynamics can be found in [1] and [9]. The progress and challenges in simulations of railway vehicle dynamics are reviewed by Evans and Berg in [10]. The topics of bogie hunting and stability assessment using computer simulations are analysed in detail in [11]. The state of the art of modelling of suspension components, model building, plausibility check and validation presented at the IAVSD Symposium in 2011 is published in [12].

Meanwhile, the reliability of simulation results is widely recognised, and the simulations allow the amount of physical testing to be reduced. The opportunity of

virtual testing and evaluation of results in accordance with the standard EN 14363 [13] is evaluated by Mazzola et al. in [14]. Also Suarez et al. [15] present the use of simulations and evaluation of results according to EN 14363 during the vehicle design and in regard to its maintenance. Wilson et al. reviewed in [16] the assessment of safety against derailment around the world using simulations as well as testing. Kuka et al. [17] present an example of modelling and simulation of tilting train and model validation by comparison of simulation results with measurement. Spiryagin et al. [18] present a proposal for modelling and simulations in regard to acceptance procedure for a locomotive model that is currently intended for Australian freight locomotives. These examples of references demonstrate importance of running dynamics simulations, which nowadays play an indispensable part in the vehicle design.

This article presents the state of the art of the application of multi-body simulations and the progress of investigations related to validation of simulation models in the context of vehicle authorisation. The typical vehicle dynamics methods and investigations used during the engineering process in the rolling stock industry are presented in Chapter 2. An overview of the possible usage of multi-body simulations in the context of vehicle authorisation in the recent standards is provided in Chapter 3. The validation of vehicle models is the crucial condition for the application of simulations in vehicle acceptance context. The recent progress of investigations related to the definition of process and criteria for a reliable model validation is discussed in Chapter 4.

# 2 Running dynamics simulations in rolling stock design

Multi-body simulation tools are used in rolling stock design and development to optimise the vehicle parameters and to conduct a wide range of investigations, including the assessment of vehicle running behaviour and the prediction of test results. The vehicle dynamics calculations undertaken during railway vehicle engineering address the following areas [1]:

- Risk assessment
- Fulfilment of customer specification
- Forecast and support of vehicle acceptance tests
- Support of other specialists during the design process.

Typical dynamics analyses used during the rolling stock design and development are:

- Eigenvalues and assessment of the running dynamics eigenbehaviour
- Running stability assessment
- Ride characteristics and comfort
- Curving behaviour
- Cross wind safety
- Simulation of stationary tests.

Ref. [1] provides descriptions concerning the aim, method, important influencing parameters, typical input data, output values as well as examples from industrial application for these analyses.

Multi-body simulations are also used for investigations of several other topics and phenomena as e.g.:

- Vehicle gauging
- Running through switches and crossings
- Wear of wheels and/or rails
- Rolling contact fatigue of wheels and/or rails
- Wheel out-of-roundness
- Rail corrugations
- Movements of bodies
- Relative displacements of suspension elements
- Forces, structural loading, cumulative load distribution
- Track access charging (in UK based on simulations)
- Longitudinal dynamics and push-pull forces in a train composition
- Interactions between vehicles (e.g. regarding gangways, buffer contact, etc.).

The state of the art paper [10] presents a more detailed overview of the applications of simulations for rail vehicle dynamics. Vehicle dynamics investigations during the product development phases, typical simulations and some challenges of running dynamics in bogie design and development are presented in [19]. Several examples of industrial applications can also be found in Ref. [9].

# **3** Application of simulations for vehicle acceptance

## **3.1 Introduction**

Multi-body simulations are increasingly used for the acceptance of running characteristics of railway vehicles in the authorisation process. Although this tendency is similar all over the world, the approaches and conditions for the application of simulations in different countries are often different.

The UK has used dynamic simulations as part of the vehicle acceptance process for many years. Although the UK is part of the EU, the separate history of the UK railways has resulted in different vehicle acceptance practices as described in Chapter 3.2.

The testing for vehicle acceptance in Europe is carried out in accordance with the standard EN 14363 [13]. The full procedure requires four test zones related to different curvatures: Straight track (test zone 1), large radius curves (zone 2), small radius (zone 3) and very small radius curves (zone 4). Each test zone is divided into test sections. Each test section has a specified length, which depends on the test zone and may also depend on the expected permissible speed for the vehicle being tested. In this method, the tests are not considered to include the worst possible track input but are considered to represent the distribution of conditions that will be encountered during normal service. The distribution of test results, such as mean and standard deviation for a normally distributed signal, are then used to calculate an estimated value to compare with the limit value.

Simulations are used in Europe primarily after modifications of an already tested and accepted vehicle or for vehicles of design very similar to already accepted vehicles. The application of simulations as proposed in UIC 518:2009 [21] and prEN 14363:2013 [22] is a replacement of a physical test by computer simulation of the same test procedure. To reproduce all aspects of the physical tests conducted in accordance with those documents, the scatter of test conditions should also be introduced in simulations and assessed using the statistical assessment method as used in EN 14363. The application of simulations instead of testing is allowed only under specified conditions as described in Chapter 3.3.

When computer simulation is used with the intention to reproduce all aspects of a physical test as is specified in prEN 14363:2013, then the main issue is to ensure that the virtual homologation process reproduces the actual dispersion of environmental and uncontrolled boundary conditions affecting the physical test for vehicle acceptance [14]. However, the advantage of simulations is the detailed knowledge of testing conditions which are inserted as simulation input parameters and therefore reduce the scatter of results of virtual testing, see Figure 1.



Figure 1: Comparison of scatter and uncertainties of physical test (top) and virtual test using computer simulation (bottom).

The procedure introduced in the USA uses the advantages of simulation against physical testing and requires simulations under clearly specified conditions and input parameters. Those simulations are required for all vehicles accepted for operation with high speeds and at high cant deficiencies prior to testing, with the aim for identification of vehicle dynamic performance, see Chapter 3.4.

A possibility to use computer simulations in place of specified physical tests is also stated in Australian standards and Rail Corporation New South Wales [18]. Other countries as Japan, China, and Korea, all allow use of simulation for parts of the vehicle acceptance process [16], although the circumstances where this is allowed vary.

The validation of the vehicle model represents the most important concern in regard to application of simulations instead of physical tests. The conditions regarding the model validation specified in the standards and the recent progress of investigations on this topic are discussed separately in Chapter 4.

## **3.2** Use of simulations in the UK

In the UK [20], the resistance of vehicles to flange climbing derailment was first determined by two quasi-static laboratory tests:

- wheel unloading on twisted track, where the wheels are jacked up to represent a severe twist input and the wheel unloading  $(\Delta Q/Q)$  measured
- bogie rotational resistance where the bogie is rotated on a special turntable and the resistance measured.

If the vehicle passes both of these tests then its flange-climbing derailment resistance is considered to be acceptable. In practice, these tests were found to be too restrictive and not appropriate for novel designs such as articulated vehicles where existing experience was not a good guide. Therefore, an alternative approach was adopted by simulation of the derailment ratio Y/Q (or L/V) at low speed in a range of test cases of different curve radii with defined exit transition geometry and a superimposed twist defect. The simulation model must first be validated against the results of the static laboratory tests.

Also in the UK, simulation has long been used to verify the track forces exerted by a vehicle as required in Group Standard GM/TT0088 [23]. This approach is considered to be much more cost-effective than the use of load-measuring wheels which traditionally have been rarely used in UK vehicle acceptance. In the lateral direction, traditionally the peak force is determined by simulation of the vehicle running over an assault course comprising a lateral track kink followed by a sinusoidal input of either fixed wavelength at varying speed or of varying wavelength at fixed speed designed to excite the sway and yaw modes of the vehicle body [24]. This aspect of vehicle behaviour is considered to be a maintenance rather than safety issue and there are no specific requirements defined for the validation of the model used in this case.

More recently, there has been extensive use of simulations to obtain derogation against the requirement of UK Group Standards to carry out a ride test on "representative" track. The test cannot be undertaken in another country because of unique features of the UK railway such as the use of 60-foot (18.3 m) jointed rail. In recent years many vehicles, particularly on-track machines, have been accepted by simulation of the UK ride test using models validated against ride testing in a different country.

Finally, simulations are also extensively used in the vehicle gauging process to ensure vehicles remain within the UK's uniquely restrictive structure gauge. The simulation models used for gauging have to have been validated against a sway test to determine vehicle movements.

## **3.3 Simulations in European standards related to vehicle acceptance**

### 3.3.1 Overview

The opportunity to apply simulations rather than physical on-track tests is considered in prEN 14363:2013 [22] for the following four applications:

- extension of the range of test conditions where the full test programme has not been completed
- approval of vehicles following modification
- approval of new vehicles by comparison with an already approved reference vehicle
- investigation of dynamic behaviour in the case of fault modes.

The evaluation of the estimated values which have to be compared with the limit values for the vehicle acceptance can be provided by:

- statistical evaluation according to conditions specified for testing, which requires simulations of the complete on-track test
- relative assessment using a combination of simulation and previous on-track tests.

When using the latter option of relative assessment, the simulated dynamic behaviour of the tested vehicle as well as the new or modified vehicle are compared under identical boundary conditions on at least 3 sections of each test zone. For every required assessment quantity, the simulation results for both new or modified vehicle and the tested vehicle have to be evaluated. The new or modified vehicle's estimated value for the assessment quantity is calculated by adding the average difference of the compared sections from one test zone to the estimated value from the test report for the tested vehicle. This new estimated value has to be compared to the limit value.

The allowed range of the application of simulation instead of testing is not related to the change of the vehicle parameters due to the modification or in comparison to the reference vehicle, but it depends on the application case and on the dynamic behaviour of the vehicle under approval. The application cases and the limits to be fulfilled are described below.

## **3.3.2 Extension of the range of test conditions**

Simulation can be used to cover the deficiencies as part of the vehicle approval. This situation could arise where

- sufficient track length is not available to meet the requirements for some zones
- the full range of speed and cant deficiency has not been tested
- the full range of wheel-rail contact conditions has not been covered
- measuring channels failed, or provided unreliable results.

The data from on-track test measurements are extended with the results from simulations for the test conditions which were not available during the test and the statistical evaluation according to EN 14363 is conducted for the complete set of data.

### 3.3.3 Approval of vehicle modification

Simulation can be used for vehicle acceptance rather than physical testing after a vehicle modification, for example:

• change of the use of the vehicle

- upgrade of the vehicle
- modifications to improve the running behaviour:
  - during or following the approval test programme
  - when some tests were done in a preliminary vehicle configuration and the final configuration is defined afterwards.

Simulations have to be carried out for all test zones to demonstrate that the performance of the new vehicle is consistent when compared to the previously tested vehicle. If a vehicle has been tested and found to exceed some of the limit values, then it is permitted to use numerical simulations to demonstrate that modifications to the vehicle will improve the behaviour sufficiently to meet the limits. The values that previously exceeded the limits have to be under the limit values for track loading and at least 10 % below the limits for running safety. At the same time all other values have to remain below the limit and not increase by more than 1/3 of the previous margin to the limit value.

### 3.3.4 Approval of new vehicles by comparison with a reference vehicle

Simulation can be used to approve acceptance of vehicles with a range of different types within the fleet. One vehicle type has to be defined as the reference vehicle, approved by measurements and its model validated against those measurements. Vehicles that are similar to the reference vehicle can then be approved using numerical simulations, which have to be conducted for all test zones. The vehicle approval is accepted if the simulations demonstrate that the performance of the new vehicle is consistent when compared to the reference vehicle. If the change to the dynamic performance results in:

- an increase in any assessment value compared to the reference vehicle
- and/or a fundamental change in the frequency and/or amplitudes of the dynamic response

then a full review has to be carried out including analysis that investigates the changes to the dynamic response(s) of the new vehicle compared to the reference vehicle. This comparison has to be carried out for at least 3 sections of each test zone. The vehicle approval using simulations is accepted, if this comparison demonstrates that:

- the assessment values for running safety from simulations do not increase by more than 1/3 of the previous margin to the limit values
- and at the same time the values for track loading from simulations do not increase by more than 2/3 of the previous margin to the limit values.

### 3.3.5 Investigation of dynamic behaviour in case of fault modes

Simulation is permitted as an assessment method to investigate fault modes as e.g. failure of yaw dampers, air suspension, tilt systems or active suspension systems. The only conditions specified for this application in [22] are that the vehicle model has to be used within its range of validity, and the validity of the simulation of fault modes have to be reviewed and confirmed as being appropriate by an independent reviewer.

## 3.4 Use of simulations for vehicle authorisation in the USA

In contrast to the use of simulations in context of authorisation in Europe to replace physical tests for particular vehicles, the application of simulations in the USA is recently requested for all vehicles accepted for operation with high speeds and at high cant deficiencies, with the aim of identifying vehicle dynamic performance issues prior to service. FRA regulation 46 Part 213 [25] describes in Appendix D entitled "Minimally Compliant Analytical Track (MCAT) Simulations Used for Qualifying Vehicles to Operate at High Speeds and at High Cant Deficiencies" the requirements for the use of computer simulations to demonstrate compliance with the vehicle/track system qualification testing requirements. These simulations have to be performed using a track model known as MCAT, which contains defined geometry perturbations at the limits that are permitted for a specific class of track and level of cant deficiency. MCAT is required to be used for the qualification of new vehicle types intended to operate at track Class 7 speeds or above (this means with maximum speed of 125 m.p.h. or higher), or at any curving speed producing more than 6 inches of cant deficiency. MCAT layout consists of nine segments, each designed to test a vehicle's performance in response to a specific type of track perturbation. The basic layout and the values of wavelength, amplitude of perturbation, and segment length, are provided in [25]. The MCAT simulations are independent of the route, and once conducted, will have examined the vehicle/track system performance under the majority of worst-case conditions that might be found on any route. To develop a comprehensive assessment of vehicle performance, simulations have to be performed for a variety of scenarios, on straight or curved track, or both, depending on the level of cant deficiency and speed (track class).

## 4 Model validation

## 4.1 What is model validation?

The American Society of Mechanical Engineers Standards Committee on verification and validation in computational solid mechanics describes model validation as a two-step process [26]:

- Verification: The process of determining that a computational model accurately represents the underlying mathematical model and its solution.
- Validation: The process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.

The verification is thus primarily dedicated to the code verification conducted by the code developers, while the validation represents the comparisons with measurements assessing the quantitative accuracy of the simulation model in regard to the intended application.

The model validation has to be carried out by the model developer and considers the particular model stage and the particular intended application of the model. The

validation consists of the quantitative comparisons of the computational and experimental results and the determination whether there is acceptable agreement between the simulation and the measurement results for the intended area of the simulations using the validated model.

The comparison with measurement used for model validation should take into account all errors and scatter of conditions influencing both measurement as well as simulation: the errors of running dynamics measurement, the errors in the measurement of track layout and track irregularities, measurement of rail profiles and wheel profiles, as well as the scatter of the test conditions as e.g. friction coefficient between wheel and rail. The validation assessment should also take into account the number of repeated tests used for validation and their reproducibility.

So far, quantitative measures to assess the validation of railway vehicle models used in context of vehicle acceptance are rarely specified, and the validation approval is matter of subjective judgement by the responsible person. Fries et al. [27] states that a model is validated when the appropriate authority declares it to be validated:

- Professor
- Rail agency chief engineer
- Government regulator.

From the European point of view, the following can be added with the top priority:

- Accredited assessor
- Officer of the railway authority.

The most difficult topic is the specification of the particular validation limit, i.e. the maximum deviation ("matching error") allowed for a successfully validated model. Even the selection of the type of this error is far from easy. A percentage (relative error) is commonly used as a limit, although investigations in the Dynotrain project showed that this is not the best approach [31] and proposed the use of absolute matching error. Moreover, a good agreement with a particular measurement does not mean that the model represents correctly the behaviour of the investigated system. Thus, it is important to compare a sufficient, statistically relevant amount of data simulation – measurement. This topic is not satisfactorily explored yet. Ref. [27] states that "validation criteria depend upon the intended use of the model and many other factors. As model validation work continues, some generally accepted criteria or guidelines may be developed, but they do not currently exist."

The discussion about the model validation in [12] considers two general validation approaches. The validation can either be realised by checking the correctness of the physical relationships in each model component, or a signal based verification can be realised by comparing the input and output data (i.e. using a "black box" model), respectively. The validation of suspension and coupling components of a vehicle model can be a part of the validation process; however, the validation of component modelling does not mean that the complete model is validated. The model validation in railway applications is often a combination of physical and signal based validation intended to approve the static, quasi-static as well as dynamic model behaviour. The models are developed in the simulation tools verified by the code developer and the model validation can be supported by the validation of component models. Experience with validation of railway vehicle models in context of vehicle acceptance process has been gained for several years in the UK and introduced in the Railway Group Standard Guidance Note GM/RC2641 [28]. The vehicle model has to be validated against the static tests and the ride test. The same model validation is incorporated as recommended practice in to the European standard EN 15273-2:2013 [29] dealing with vehicle gauging.

Comparisons of simulation and ride test results represent an important part of model validation, as this is the best possibility to assess the validity of dynamic performance of the vehicle model. UIC 518:2009 [21] as well as prEN 14363:2013 [22] propose that the validation of vehicle models should be conducted using the on-track tests specified for testing of running characteristic. Unfortunately, neither document contains a specification of the allowable differences between simulation and test results. Because of the lack of quantitative criteria, an assessment by an independent reviewer is required to ensure that the model provides a sufficient representation of reality for the intended application.

An evaluation of subjective assessments by engineering judgement was carried out as a part of investigations in the Dynotrain project [31], [32], [33]. Time or distance plots were assessed using simple "Yes/No" method by project partners as well as during a workshop with 26 attendees (professors for railway vehicle dynamics, experts from industry, railway companies, testing and research institutes, members of standardisation committees and Dynotrain project partners). The assessments demonstrated that the presentation of the provided measurement and simulation results influences the reviewers' assessments. An example in Figure 2 shows the percentage of positive answers provided by workshop attendees regarding two plots displaying the same data using different scale of the vertical axis.



Figure 2: Effect of plot scale on the satisfaction in regard to model validation [32].

Moreover, the workshop results showed that the final assessment of each single comparison depends on the reviewer's "strictness". This "strictness" of each particular reviewer remained rather the same in all categories of plots and was related neither to the reviewer's affiliation nor experience. Although the assessments carried out during the workshop were related to single diagrams only, without any background information about the vehicle type, test conditions and simulation procedure, and thus cannot be considered as representative validation assessments, they illustrate the weakness of subjective judgements of time or distance diagrams. Publications about the experience with validation of vehicle models against the ontrack tests according to EN 14363 are rare, although the specialists working on simulations probably use those tests to validate their model. Two trials were initiated by UIC during the preparation of the revision of UIC 518 from 2009 [21]; the experience with one of them is published in [30]. A unique, comprehensive work on this topic has recently been provided in the framework of the European research project Dynotrain [31], [32], [33]. The following chapters review in detail the validation using stationary tests (4.2), validation in frequency domain (4.3) and in time domain (4.4).

## 4.2 Validation using stationary tests

Stationary tests are used to test the features of a vehicle that make it susceptible to flange-climbing derailment. Furthermore, the vehicle gauging assessment is based on the vehicle body sway test. The stationary tests have the advantage of being relatively inexpensive and repeatable.

In the UK, a model which is used for simulation of slow-speed derailment will be required to be validated against the following tests [28]:

- Weighbridge tests to verify vehicle masses and their distribution
- Wheel unloading test to verify the wheel unloading on twisted track, which depends on primary and secondary suspension vertical roll stiffness, anti-roll bars (where used) and bogie frame and car body torsional stiffness.
- Bogie rotational resistance to verify the resistance to rotation of the bogie relative to the car body, which depends on the longitudinal shear properties of the secondary springs or in freight vehicles the friction properties of the centre pivot and sidebearers.
- Sway test this is not mandated for vehicle running acceptance, but is required for models used for vehicle gauging.

If models are to be used for simulation of a ride test, it would normally be expected that they would be validated against the stationary tests as well as dynamic test results.

The model validation as used in the UK has been introduced in the recommended practice for the dynamic vehicle gauging method in EN 15273-2:2013 [29]. The dynamic gauging is based on the assessment of vehicle movements by computer simulation and comparison with measured infrastructure cross profile sections. The vehicle model applied for the dynamic gauging has to be validated using the weighbridge test, bogie rotational resistance test, sway test and dynamic ride test. To assess the accuracy of simulation model, the so called matching errors are calculated:

- For sway test validation: absolute matching error
- For validating other static tests: percentage matching error.

The matching errors are compared with the matching error limits (for both maximum and average errors) which are defined for the following quantities:

- Wheel loads (individual, wheelset load, side loads)
- Bogie rotational resistance X-factor (see definition in [13])

• Primary roll angle, secondary roll angle, solebar sway and solebar drop (solebar is defined as the outer edge of the underframe at floor height, see [29]).

The recent draft prEN 14363:2013 [22] proposes the following stationary tests (i.e. static and slow speed tests) to be used to validate different aspects of the vehicle model:

- Measurement of vertical wheel forces (weighbridge)
- Twist test (wheel unloading test)
- Bogie rotation test
- Sway test or roll coefficient measurement.

Another static test suited to support the model validation is the lateral resistance test, which means the measurement of lateral force-deflection of the suspension when mounted in the vehicle. Such test can be performed moving the vehicle body in lateral direction relative to the running gear [22].

Identification of eigenfrequencies and eigendampings of the rigid body movements can be conducted by so called wedge test. However; this test is not often used because of the costs and limitations of this method especially for articulated vehicles. A recent paper by Rosenberger et al. [34] presents an approach to validate a railway vehicle model by means of this wedge test. A special emphasis is placed on the Ibrahim Time Domain Method, which is a comprehensive method to identify the vibration characteristics of dynamical systems. The paper shows that wedge tests in real-world and in simulation in combination with the analysis with the Ibrahim Time Domain method enable an identification of a high number of the eigemodes of the rigid body movements as well as vehicle body structural eigenmodes of railway vehicles. The number of identified eigenmodes is mainly limited due to the fact that even the different arrangements of wedges represent always a more or less vertical excitation.

Comparisons with the stationary tests can support the identification of unknown or verification of uncertain vehicle parameters. Weighbridge data can be used to check and to adjust the vehicle mass and the position of the centre of gravity in horizontal plane. The twist test can check the primary vertical suspension stiffness. The stiffness of primary and secondary lateral suspension can be checked using the lateral resistance test. The bogie rotational resistance test can check the longitudinal stiffness of the secondary suspension and the friction properties of the car body to bogie connection of freight vehicles. Sway test can support the identification of the vertical position of the car body centre of gravity and of the roll stiffness of secondary suspension.

Although the stationary tests provide several opportunities to verify and justify model input parameters, this aim cannot always be fully achieved. Deviations between the results of stationary test measurement and simulation often cannot be clearly related to one parameter (e.g. the difference between measurement and simulation of a sway test can be related either to the vertical position of car body centre of gravity or to the suspension stiffness). Moreover, problems and pitfalls may be encountered using the comparisons with the stationary tests for model validation as illustrated on the following examples (see also [35]).

### • *Difficulty* - *Wheel load measurement:*

Usually, measured wheel loads can be matched quite easily in a vehicle model by using appropriate masses and ensuring appropriate positioning of the centres of gravity. However in some cases the vehicles may have inherent twist defects or errors in suspension setup which give uneven load distributions. It is possible to represent these effects in a vehicle model, and if a specific vehicle is being modelled, for example after a derailment, this is essential. However, for vehicle acceptance, it will not be appropriate unless these errors are representative of the entire fleet, and it would be advisable to have the errors fixed before further testing and validation is carried out.

### • Difficulty - Wheel unloading test:

A common practical difficulty is to ensure that the vehicle is fully level at the start of a wheel unloading test. However, even if the vehicle is not fully level at the start of the test, provided that full load-displacement hysteresis loops have been recorded during the test at each wheel, it is possible to estimate the actual twist conditions during the test and to recreate them in the simulation.

### • Difficulty - Bogie rotational resistance test:

In principle, the simulation of rotational resistance is straightforward. A simulation input is devised which rotates the bogies at a known rate to a given angle, and the yaw torque required is measured. For vehicles with yaw dampers the resistance varies according to the speed of rotation, and the validation must be demonstrated for a range of rotational speeds.

In practice, the biggest problem is accounting for friction in the measuring rig. This will typically be load-dependent and has to be determined by making a measurement with a dummy load corresponding to half the vehicle weight. The frictional resistance in the rig then can be subtracted from the measurement with the vehicle in place.

The presented examples show that the validation comparisons sometimes may alter from a model justification to a justification and correction of the measurement inexactness.

Uncertainties calculation of a model parameters and error propagation is investigated by Aizpún et al. [36]. This method is virtually validated using a model of an Inter-city train operated in Spain. The chosen stationary test was the wheel unloading test, which is one of the most common stationary tests.

The effect of the model adjustments and validation using stationary tests on the agreement achieved in simulations of the on-track tests were recently extensively investigated in the Dynotrain project. The results showed that the model improvement using stationary tests in regard to simulation of the on-track test is often marginal if reliable vehicle model data is available [33]; see Chapter 4.4.3. Moreover, the model results can be even worse than before the model adjustment. The Dynotrain investigations resulted to a conclusion, that a comprehensive comparison with the on-track tests is indispensable for reliable validation of a vehicle model intended for the simulation of on-track test.

### 4.3 Validation by comparisons in frequency domain

The comparison of the frequency content of simulation and measurement results is a part of the model validation. Some hints and examples of good and insufficient agreement are shown in [21], [22]. An informative analysis regarding this topic is provided by Fries et al. in [27]. The authors compare the effect of the applied type of frequency domain calculation, i.e. linear spectrum (LS) versus power spectral density (PSD), and the form of plot's axes, i.e. linear versus semilog axes. The PSD magnitude agreement is generally poorer than the LS agreement. In general, the percentage error for a PSD comparison is about twice as large as the percentage error for an LS comparison.

Some researchers prefer to plot spectral results on semilog axes rather than linear axis. An observer not aware of the differences in views might think that the results presented in a diagram with semilog axis show better agreement, see examples of car body lateral acceleration spectral comparisons in Figure 3. The lateral accelerations were measured and simulated at both ends of the car body. The accelerations were averaged to produce a car body lateral acceleration at the centre of gravity. A PSD presented in a diagram with semilog axis shows better amplitude agreement than an LS with the linear axis. A semilog plot of the LS would appear even better to a casual observer.



Figure 3: Effect of the type of frequency domain diagram and axis scaling on the comparison simulation – measurement (from [27]).

When frequency domain analysis is used to compare resonant vehicle responses and their corresponding predictions, the authors of Ref. [27] recommend that frequencies should agree within 10 to 15 percent or less for nearly all resonant responses. A few responses with larger differences should not necessarily disqualify a model from validation. Amplitudes of linear spectra for predicted and measured resonant responses should agree within 10 to 15 percent or less for nearly all resonant responses. Amplitudes of PSD spectra should agree within 20 to 30 percent. A few responses with larger differences should not necessarily disqualify a model from validation.

### 4.4 Validation by comparisons with on-track tests

#### 4.4.1 Introduction

There are almost no limit values specified for the comparisons between simulation and measurement in the time domain results from the on-track (ride) test so far. One possibility to assess time histories is an application of validation metrics as discussed in Chapter 4.4.2. The validation approach recently developed in the Dynotrain project using values based on EN 14363 is presented in Chapter 4.4.3.

### 4.4.2 Validation metrics

Agreement between measurement and simulation regarding the time histories is traditionally carried out by engineering judgement. The validation metrics represent quantitative measures introduced with the intention to provide results in agreement with a subjective assessment by engineering judgement [37].

One of the metrics used to compare the time or distance domain diagrams is the integral approach described by magnitude, phase and comprehensive error factors, proposed by Sprague and Geers in [38]. Small values of error factors represent good agreement. By using the same sampling rate and the same length of time or distance interval for the compared measurement and simulation signals, the definitions of error factors proposed in [38] can be expressed by the following formulae [39].

• Sprague and Geers magnitude error factor:

$$M_{SG} = \sqrt{\frac{\sum_{i=1}^{n} c_i^2}{\sum_{i=1}^{n} m_i^2}} - 1$$
(1)

with  $c_i$  - simulated values

 $m_i$  - measured values.

• Sprague and Geers phase error factor:

$$P_{SG} = \frac{1}{\pi} \cos^{-1} \left( \frac{\sum_{i=1}^{n} c_i m_i}{\sqrt{\sum_{i=1}^{n} c_i^2 \sum_{i=1}^{n} m_i^2}} \right)$$
(2)

• Sprague and Geers comprehensive error factor:

$$C_{SG} = \sqrt{M_{SG}^{2} + P_{SG}^{2}}$$
(3)

Fries et al. [27] assess the experimental uncertainty first, to ensure that the requirements on the model validation are not as strict or even more strict than the requirements on the repeatability of testing. They present results for a test repeated 4 times. The  $2^{nd}$ ,  $3^{rd}$  and  $4^{th}$  test were compared with the first test run and showed comprehensive error factor by Sprague and Geers between 0.10 - 0.32. The investigations presented in [27] lead to conclusion that the comprehensive metrics of a validated model can range from around 0.1 to greater than 0.5 for different responses of the same model. The existence of some values above 0.5 should not necessarily disqualify a model for validation.

The investigations of metrics by Sprague and Geers in Dynotrain project identified three kinds of drawback in regard to simulations of vehicle acceptance tests [31]:

- The magnitude error factor is a relative measure, which at low measured values often contradicts the experts' assessments: The magnitude error factor is high in spite of a positive subjective assessment.
- Small frequency deviation between simulation and measurement or a poor signal synchronisation strongly influences the phase error factor, while subjective experts' judgement often disregards effects like this: The phase error factor is high in spite of positive subjective assessment.
- The magnitude error factor is dominated by the signal's quasi-static level. Thus, the bigger the signal's static level, the lower is the influence of the dynamic oscillations superposed over this static level. In such a case, the magnitude error factor is low although the subjective assessment is negative due to poor agreement of dynamic oscillations.

#### 4.4.3 Validation approach proposed as the result of Dynotrain project

The investigations conducted in the European research project Dynotrain resulted in a proposed new approach regarding the method, criteria and limits for the validation of multi-body vehicle models used in simulations of on-track acceptance tests [31], [32], [33]. These investigations represent unique work in regard to both simulations and measurements. The analyses were carried out using measurements with a test train consisting of several different vehicles and using 10 force measuring wheelsets, running over 20 days through four European countries and being equipped with a simultaneous recording of track irregularities and rail profiles. The simulations included a total of 10 vehicle models prepared by different partners using three multi-body simulation tools. The comparisons with measurements conducted in Dynotrain were carried out for all vehicles in the same 17 selected track sections, called validation exercises. The following validation assessments between the simulation and the measurement results were evaluated and compared:

• Assessment based on quantities filtered and processed by analogy with EN 14363 (evaluated in the part of track section with constant curvature)

- Subjective engineering assessment using a simple "Yes/No" method by project partners as well as during a project workshop with invited experts
- Validation metrics, i.e. quantitative measures comparing simulation and measurement in the time histories.

An example of validation exercises in Figure 4 shows comparison between simulation and measurement of the guiding force on the outer leading wheel at two vehicles tested in Dynotrain project:

- An empty two axle freight wagon of a wagon unit Laas modelled by Alstom
- DB passenger coach Bim modelled by Bombardier Transportation.

The figure presents time (distance) diagrams, the values evaluated in analogy with EN 14363, the error factors of validation metric by Sprague and Geers as well as the result of subjective assessments by project partners.



Figure 4: Example of validation exercise from Dynotrain project: Test run in Germany, curve radius 282 m, speed 68 km/h (from [31]).

The variations of model parameters, track and wheel/rail contact geometry data resulted in to a total of 78 model configurations. The comparisons between simulation and measurement using values based on EN 14363 represented more than 50 000 single comparisons used in the evaluation.

The process which was used to evaluate the final proposal for model validation is shown in Figure 5. The evaluation showed that the subjective assessment is dependent on the strictness of the individual reviewer. The use of error factors of validation metrics did not provide sufficient contribution towards an objective and reliable validation. The final proposal is thus based on the comparison of data evaluated in analogy with EN 14363.

The partners were asked to propose the validation limits for the values evaluated in analogy with EN 14363 according to their experience. These validation limits were then applied on the 78 model configurations and adjusted based on the feedback about the overall validation results achieved.

The proposed validation method consists of analysing 12 quantities shown in Table 1, filtered and processed by analogy with EN 14363. They cover the quasi-static and dynamic wheel/rail force measurements and vertical as well as lateral car body accelerations.



Figure 5: Evaluation of validation criteria and limits in Dynotrain [33].

They have to be compared on at least 12 test sections containing sections from all 4 test zones according to EN 14363; at least 3 sections from each test zone. Each quantity is evaluated using at least two signals, e.g. the vertical acceleration above the leading and the trailing bogie; thus at least 24 simulated values are compared with their measured counterparts for each quantity, see Figure 6. For the maximum value calculated as 0.15% or 99.85%-value, the higher magnitude of the 0.15%- and 99.85%-values (absolute value) is used. The quasi-static values are calculated as 50%-values (medians). They are applied with their sign to check the agreement of both magnitude and direction of those quantities.

The difference  $D_v$  between the simulated value  $S_v$  and the corresponding measured value  $M_v$  is evaluated for each value and each quantity. This difference  $D_v$  is transformed depending on the sign of the measured value  $M_v$  so that, if the magnitude of the simulation value is higher than the magnitude of the measurement (simulation overestimating the measurement), the difference is positive, and vice versa.

The following values shall be calculated for the whole set of differences  $D_v$  between the simulation and measurement for each quantity (e.g. for all  $Y_{qst}$  values) and compared with the validation limits:

- Mean value
- Standard deviation.

Table 1 shows the proposed validation limits (matching errors) for the standard deviation of the differences between simulation and measurement. The validation

limits for the mean values are equal to 2/3 of the limits for the standard deviation shown in Table 1. The validation limits for accelerations (mean as well as standard deviation) for freight vehicles or vehicles without secondary suspension are twice the relevant limits for other vehicles.

Quantity	Notation	Unit	Filtering	Validation limit for standard deviation
Quasi-static guiding force	Y <sub>qst</sub>	kN	Low-pass filter 20 Hz	5
Quasi-static vertical wheel force	$Q_{ m qst}$	kN	Low-pass filter 20 Hz	$4 (1+0.01 Q_0)$ $Q_0$ - static wheel load [kN]
Quasi-static quotient Y/Q	( <i>Y/Q</i> ) <sub>qst</sub>	-	Low-pass filter 20 Hz	0.07
Quasi-static sum of guiding forces	$\Sigma Y_{qst}$	kN	Low-pass filter 20 Hz	6
Guiding force, maximum	Y <sub>max</sub>	kN	Low-pass filter 20 Hz	9
Vertical wheel force, maximum	$Q_{\max}$	kN	Low-pass filter 20 Hz	$6 (1+0.01 Q_0)$ Q <sub>0</sub> - static wheel load [kN]
Quotient <i>Y/Q</i> , maximum	(Y/Q) <sub>max</sub>	-	Sliding mean (2 m window)	0.10
Sum of guiding forces, maximum	$\Sigma Y_{\rm max}$	kN	Sliding mean (2 m window)	9
Car body lateral acceleration, rms	$\ddot{y}_{rms}^{*}$	m/s <sup>2</sup>	Band-pass filter 0.4 to 10 Hz	0.15
Car body vertical acceleration, rms	$\ddot{z}^*_{ m rms}$	m/s <sup>2</sup>	Band-pass filter 0.4 to 10 Hz	0.15
Car body lateral acceleration, max.	$\ddot{y}_{\max}^*$	m/s <sup>2</sup>	Band-pass filter 0.4 to 10 Hz	0.40
Car body vertical acceleration, max.	$\ddot{z}^{*}_{\max}$	m/s <sup>2</sup>	Band-pass filter 0.4 to 10 Hz	0.40

Table 1: Quantities compared and validation limits proposed in regard to simulation of on-track test [31].



Figure 6: Validation approach from Dynotrain project: Differences between simulated and measured values (left) and calculation of their mean and standard deviation (right) [31].

The advantage of the proposed method is a comparison of a large set of data. This allows checking of the agreement of the overall vehicle dynamic behaviour between the real vehicle and the simulation model. The weakness of the model in question can be identified by displaying the mean and the standard deviation of differences between simulation and measurement normalised by the corresponding validation limit (see example in Figure 7). A vehicle model is then validated if the absolute magnitudes of all normalised values are not higher than 1.

The Dynotrain project also investigated the effect of the knowledge of measured track parameters and the effect of the model adjustments by comparisons with stationary tests on the validation results of the on-track tests. An example of the latter effect is shown in Figure 7. This figure presents simulation results conducted by CAF regarding the high speed train supplied by this company for TCDD (Turkey). The on-track test results used for the validation of this model were provided by CAF, whereas the track irregularity data and the actual rail profile data were not available. The validation results are shown for three model configurations:

- A1 initial model before the comparisons and adjustments using stationary tests, but considering the measured static wheel loads
- G1 model after adjustments using the twist test (wheel unloading test)
- O1 model G1 further adjusted by comparisons with bogie rotational resistance test, roll coefficient measurement and sway test.



Figure 7: Effect of model adjustments by stationary tests on the validation of the ontrack test (see explanation in text); from [33].

As can be seen, the model improvements by comparisons with the stationary tests are rather marginal; occasionally, the results for some quantities are even worse than before the adjustment. It is believed that this rather negligible model improvement by stationary tests is due to well known model parameters of this recently developed vehicle.

## 5 Summary and outlook

Multi-body simulations developed from routines used by researchers and engineers to now mature and reliable programmes used in powerful simulation packages. They are used by vehicle and component manufacturers, operators, infrastructure companies, consultants and engineering service providers, research institutes and universities.

The majority of recently issued standards and guidelines as well as standards in preparation consider the usage of simulations as a part of the vehicle authorisation process, although the circumstances where this is allowed vary. This article presents the recent status of the conditions for usage of simulations in the authorisation of railway vehicles in Europe and in the USA.

A fundamental concern for the usage of computer simulations is the validation of multi-body vehicle model. The article provides an overview about the experience with model validation against static tests and dynamic ride (on-track) tests as well as the results of recent investigations carried out by Transportation Technology Center, Pueblo, USA and in the framework of the European research project Dynotrain. Although there is no common validation method and quantitative limits to demonstrate the validity of vehicle models, the investigations presented provide significant progress in this topic. The new approach presented from the Dynotrain project provides a specification of quantities, criteria and limits to be used for model validation approval in regard to simulation of on-track tests. Application of this proposed validation method on further vehicle models and further investigations and development of quantitative measures as e.g. the error factors of validation metrics will further improve the objectivity of the model validation and ensure credible and reliable replacement of physical tests by computer simulation.

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