# SOLUTION OF WHEEL-RAIL CONTACT FORCES SUITABLE FOR CALCULATION OF RAIL VEHICLE DYNAMICS

Polách, O.[+]


## SYNOPSIS

In the paper a simple and fast solution of wheel-rail contact forces is presented. The analytically solution provides the value of tangential contact forces for various longitudinal and lateral creepages and spin. The contact forces value can be derived from relatively few experimental results or it can substitute Kalker's solution.


## INTRODUCTION

The question of wheel-rail rolling contact has always been important because the knowledge of the contact forces acting in the contact is necessary for calculating the dynamic behaviour of rail vehicles. The most popular method for the solution of the problems of vehicle dynamic is a computer simulation. It is important to find a fast solution of the rolling contact because the calculation is repeated many times. One case of such solution will be shown. This solution can be used as a general mathematical model of experimental results as well as a fast substitution for the theoretical solution according to Kalker [2], [3].

## THEORY OF WHEEL-RAIL ROLLING CONTACT

The method of solution of rolling contact forces assumes the ellipsoidal contact area and normal stress distribution according to Hertz. The maximum value of tangential stress at any arbitrary point is:

$$\tau_{max} = f \cdot \sigma \qquad (1)$$

where: $f$ - the coefficient of friction,
$\sigma$ - the normal stress.

The coefficient of friction is constant in the whole contact area. The contact area is divided into an area of adhesion and an area of slip. The tangential stress acts against the creep or slip. In the area of adhesion its value grows linearly with the distance from the leading edge (Fig. 1). In the area of slip the tangential stress reaches its maximum value according to (1).

The tangential force is determined as follows

$$F = \int_{(U)} \tau \, dx \, dy \qquad (2)$$

[+]Technical University of Transport and Communications, Žilina, Czechoslovakia, 01026

where: U - area of contact.
For directionally oriented values, we get

$$F_i = F \frac{s_i}{s} , \qquad i = x, y , \tag{3}$$

where slip

$$s = \sqrt{s_x^2 + s_y^2} . \tag{4}$$

Freibauer [1] has solved the creep-force law without spin analytically. The ellipsoid of tangential stress is transformed to a hemisphere by using the formulae:

$$y^* = \frac{a}{b} y , \qquad \tau^* = \frac{\tau}{\tau_0} a \tag{5}$$

where: $y^*, \tau^*$ - the transformed coordinates y, z
$\tau_0$ - the maximum stress in the center of the contact area.

In the area of adhesion the value of tangential stress is proportional to creep. This creep is proportional to slip s and the distance from the leading edge (Fig. 1). The ratio

$$\varepsilon = \frac{\tau}{a \sqrt{1 - y^2/b^2} - x} = -\tan \gamma \tag{6}$$

determines the gradient of tangential stress in the area of adhesion. The value of $\varepsilon$ according to [1] is

$$\varepsilon = \frac{2}{3} \frac{C.\pi.a^2.b}{Q . f} s . \tag{7}$$

where: Q - wheel load.
C - the constant characterizing the elasticity of contact of the bodies.

The transmitted tangential force is

$$F = \iint_{(U)} \tau \, dx \, dy = \frac{\tau_0}{a} \frac{b}{a} \iint_{(U)} \tau^* \, dx \, dy^* . \tag{8}$$

The transformed tangential stress $\tau^*$ in the section parallel to the longitudinal axis is in the area of adhesion

$$\tau^* = - \varepsilon ( l - x ) , \tag{9}$$

where: l - a half of the section length $l = \sqrt{a^2 - y^{*2}}$
In the area of slip

$$\tau^* = - \sqrt{l^2 - x^2} . \tag{10}$$

Let h be the coordinate of the boundary point of the area of adhesion and slip, then

$$h = l . \delta , \tag{11}$$

where

$$\delta = \frac{\varepsilon^2 - 1}{\varepsilon^2 + 1} . \tag{12}$$

According to the theory of Hertz

$$\tau_0 = \sigma_0 . f = \frac{3 . Q}{2 . \pi . a . b} f \tag{13}$$

where: $\sigma_0$ - maximal normal stress in the contact area.
The solution of (8) according to [1] is (using (5) and (13))

$$F = - \frac{2 . Q . f}{\pi} \left( \frac{\varepsilon}{1 + \varepsilon^2} + \arctan \varepsilon \right) , \tag{14}$$

When solving the wheel-rail contact problem the spin is of a considerable importance. Spin is rotation about the vertical axis z caused by wheel conicity. Further under the title spin we will understand the relative spin which means the angular velocity about z-axis divided by speed v. There is

$$\Phi = \frac{\omega . \sin \beta}{v} = \frac{\sin \beta}{r} , \tag{15}$$

where: $\omega$ - angular velocity of wheel rolling
$\beta$ - angle of the contact surfaces
r - wheel radius.

In the following part a force effect of the spin will be mentioned. The moment effect of spin as well as the moment effect caused by lateral slip are in this case neglected because they are too small in comparison with other moments acting on the vehicle. Solution of the spin effect is more complicated. The centre of rotation is situated on the longitudinal axis of the contact area, but its position is dependent on the equilibrium of the forces and is unknown at the beginning of the solution.

Let us mention at first the case, when the longitudinal halfaxis is too small. In the limit $a \rightarrow 0$, the centre of spin rotation is positioned in the middle of the longitudinal axis, that is in the origin of the coordinate system. After the substitution of the transformation (5) the lateral force caused by the spin is

$$F_y = \iint_{(U)} \tau_y \, dx \, dy = \tau_0 \frac{b}{a^2} \iint_{(U)} \tau_y{}^* \, dx \, dy^* . \tag{16}$$

Spin causes a tangential stress perpendicular to the position vector which is going from the origin. On the longitudinal section parallel to the axis x tangential stress $\tau_y$ grows linearly with the distance from the y-axis (in both directions). Then

$$\tau_y{}^* = \tau_{yt}{}^* . x / a \tag{17}$$

where $\tau_{yt*}$ is the transformed stress, given by the formula (9) in the area of adhesion, or (10) in the area of slip. The formulae (11) and (12) are valid. After substitution (17) into (16) we get

$$F_y = -2.\tau_0 \frac{b}{a^2} \int_0^a \left[ \int_l^h |\varepsilon|.(1-x) \frac{x}{a} dx + \int_h^{-l} \frac{x}{a} \sqrt{l^2 - x^2}\, dx \right] dy* \tag{18}$$

The solution using the formula (13) is

$$F_y = 9.Q.f.K_M / 16 , \tag{19}$$

where

$$K_M = |\varepsilon|.(\frac{\delta^3}{3} - \frac{\delta^2}{2} + \frac{1}{6}) - \frac{1}{3}\sqrt{(1-\delta^2)^3} \tag{20}$$

$\varepsilon$ is given with (7) and creep s is given as

$$\begin{aligned} s_{yC} &= s_y + \Phi.a \quad \text{for} \quad |s_y + \Phi.a| > |s_y| , \\ s_{yC} &= s_y \quad \text{for} \quad |s_y + \Phi.a| \le |s_y| . \end{aligned} \tag{21}$$

The described solution has been made for the pure spin under the assumption that the longitudinal halfaxis of the contact ellipse is too small ($a/b \rightarrow 0$). In fact the centre of spin need not be in the coordinate origin. Its position on the x-axis is unknown at the beginning of the solution and the dependence cannot be solved analytically. A detailed solution is given by Kalker [2], [3] using an iterative method. This solution shows that with an increasing relation a/b the force effect of the spin grows. Looking for a fast solution to be used in the simulation calculations instead of an iterative method, a correction of dependence (19) was found.

When calculating the wheel/rail contact forces under the simultaneous presence of longitudinal and lateral slip and spin, the forces caused by longitudinal and lateral creepages and the lateral force caused by spin are calculated separately. In the equations (3), (4) and (7) instead of the slip s there is resulting slip $s_C$

$$s_C = \sqrt{s_x^2 + s_{yC}^2} , \tag{22}$$

where $s_{yC}$ is obtained by (21).
The resulting force effect in a lateral direction is given as the sum of both above described effects as follows:

$$F_{yC} = F_y + F_{yS} , \tag{23}$$

where: $F_{yS}$ - increase of the tangential force caused by the spin.

Its value is

$$F_{yS} = \frac{9}{16} a.Q.f.K_M.\left[ 1 + 6,3.(1 - e^{-a/b}) \right] \frac{\Phi}{s_C} , \tag{24}$$

where $\varepsilon$ (7) is actually given as

$$\varepsilon_S = \frac{2 \,.\, C \,.\, \pi \,.\, a^2 \,.\, b \,.\, s_{yC}}{3.Q.f.\left[ 1 + 6,3.(1 - e^{-a/b}) \right]} \tag{25}$$

and $K_M$ is obtained by (20).

The question remains what is the value of the contact stiffness C. There are two ways of answering it. The former is to use the experimental results, the latter is to use Kalker's theoretical solution and to use the proposed solution as a quick and simple substitution. Both methods are described in the following chapters.

## SOLUTION BASED ON KALKER'S RESULTS

In this case the value of C is derived by assuming an identical linear part of the creep-force law. According to the proposed theory, in the case when the creep is close to 0 ($\varepsilon \rightarrow 0$), without spin

$$F = - 8.a^2.b.C.s/3 \tag{26}$$

according to Kalker (linear theory)

$$F = - G.a.b.c_{jj}.s \tag{27}$$

where: $c_{jj}$ - Kalker's constant ($c_{11}$-longitudinal direction, $c_{22}$ - lateral direction)
G - modulus of rigidity.

After the comparison of (26) with (27) we have

$$C = 3 \,.\, c_{jj} \,.\, G \,/\, ( 8 \,.\, a ) \tag{28}$$

Because $c_{11} \neq c_{22}$, constant $c_{jj}$ will be obtained as follows

$$c_{jj} = \sqrt{\left( c_{11} \frac{s_x^{\,2}}{s} \right)^2 + \left( c_{22} \frac{s_y^{\,2}}{s} \right)^2} \tag{29}$$

The lateral force caused by the spin (19) is for $\varepsilon \rightarrow 0$ according to the proposed theory

$$F_y = - \frac{9}{16} Q.f \frac{2}{3} \varepsilon_S = - \frac{1}{4} \pi.a^3.b.C_S.\Phi \tag{30}$$

and according to Kalker's linear theory

$$F_y = - c_{23}.G.\Phi.(a.b)^{3/2} \,. \tag{31}$$

After the comparison is

$$C_S = \frac{4 \,.\, c_{23} \,.\, G}{\pi \,.\, (a.b)^{1/2}} \tag{32}$$

The comparison of Kalker's solution and described method was made in the non-dimensional coordinates. The results are shown in Fig. 2 and 3. The results of Kalker's theory was calculated by the program FASTSIM [3].

## SOLUTION BASED ON THE EXPERIMENTAL RESULTS

By using the proposed method based on the experimental results we use the characteristical values of creep-force curves: the maximum of traction force $F_{max}$, the slip $s_{max}$ in which the maximum of adhesion force is acting, and estimate of the coefficient of kinetic friction $f_k$. We assume different coefficient of friction in the area of adhesion (static friction coefficient $f_s$) and in the arrea of slip (kinetic one - $f_k$). The equation (14) with the ratio

$$k = f_s/f_k \tag{33}$$

has then the form

$$F = -\frac{2.Q.f}{\pi}\left[\frac{k.\varepsilon}{k^2+\varepsilon^2} + \arctan\varepsilon\right] \tag{34}$$

The derivative of the equation (34) by $\varepsilon$ is, after rearrangement, a quadratic equation with the solution

$$\varepsilon^2 = \frac{k}{2.(k-1)}\left[(k^2+2.k-1) + \sqrt{k^4+8.k^3+2.k^2-8.k+1}\right] \qquad k \neq 1 \tag{35}$$

We substitute into (34) the equation for $\varepsilon$ (35), then $F_{max}$ for F, and the value of $f_k$. By solving this non-linear equation (with a suitable numerical method) we get a relation of the friction coefficients k. By substituting k in eq. (35) we calculate $\varepsilon_{max}$. The value of the constant C in (7) is then

$$C = \frac{3\,.\,Q\,.\,f_k\,.\,\varepsilon_{max}}{2\,.\,\pi\,.\,a^2\,.\,b\,.\,s_{max}} \tag{36}$$

The comparison of the experiments on the roller rig with the results of the solution for the described parameters $F_{max}$, $s_{max}$, $f_k$ of the longitudinal creep-force characteristic, is shown in Fig. 4. The experiments were carried out in the Department of Rail Vehicles, Engines and Hoists of the Technical University of Transport and Communications in Žilina. The measurements were done on a test roller rig in a 1:5 scale. The model wheelset is driven by an electric motor and the track rollers are braked electrodynamically. A detailed description and discussion of the results can be found in [4].

## References


1. Freibauer, L., Adheze kola vozidla na dráze (in Czech), in "Proceedings of the VII. Scientific Conference of Technik of Transport", Editor Technical University of Transport and Communications, Žilina, 1983, pp. 214-219
2. Kalker, J. J., Veh. Syst. Dyn. Vol. 8.(1979), pp 317-358
3. Kalker, J. J., Veh. Syst. Dyn. Vol. 11.(1982), pp. 1-13
4. Polách, O., Styk kola s kolejnicí z hlediska silových poměrů v podélném a příčném směru (in Czech), Doctoral Thesis, Editor Technical University of Transport and Communications, Žilina 1983

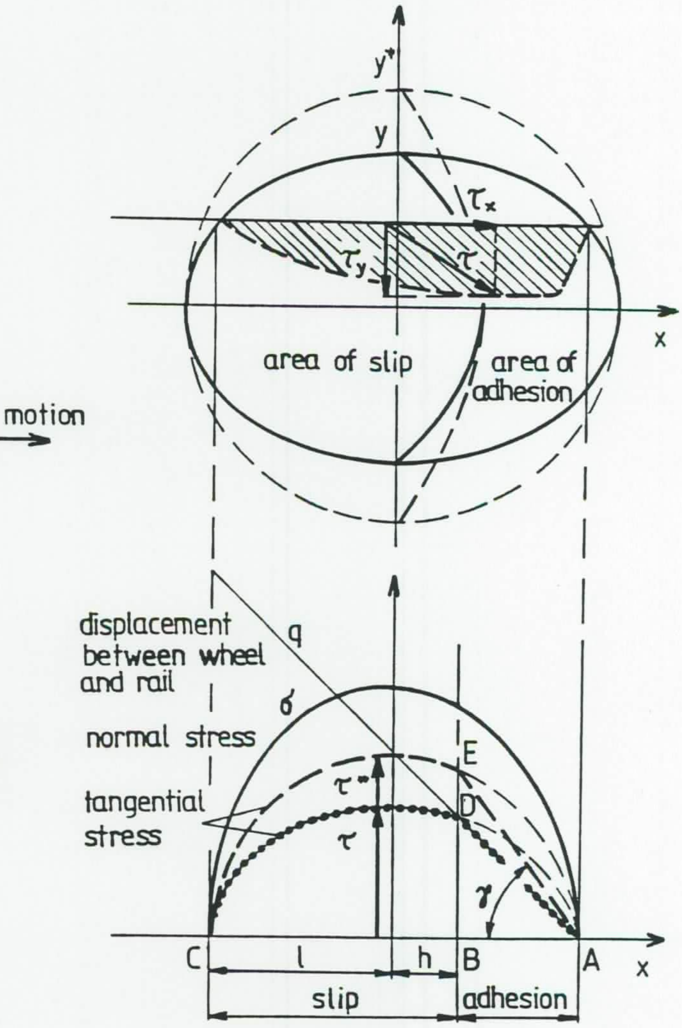


Fig. 1.
Distribution of the normal and tangential stresses in the wheel-rail contact area

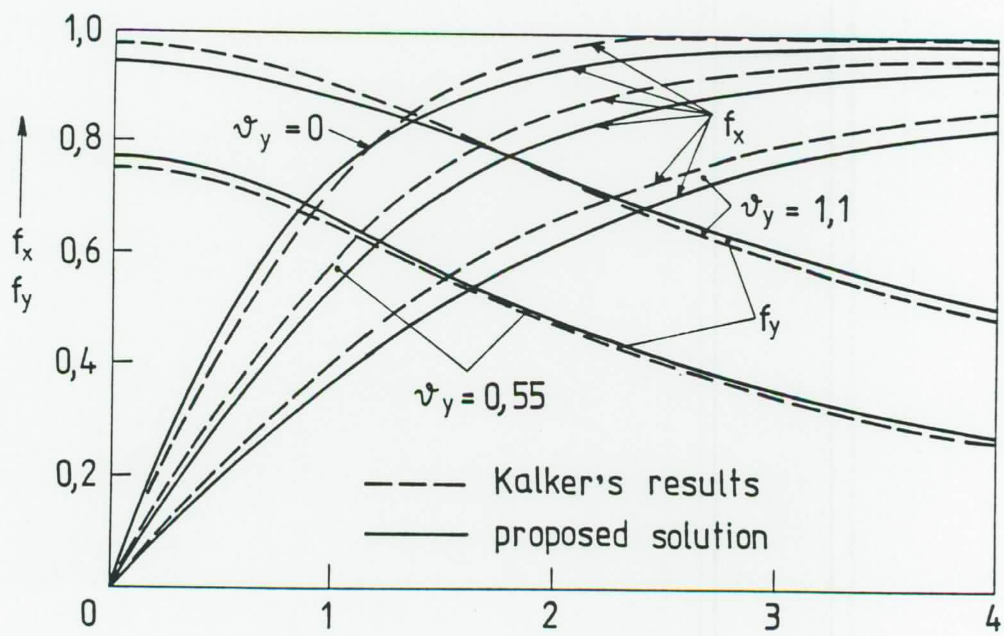


Fig. 2.
Comparison of longitudinal and lateral non-dimensional forces according to Kalker and according to the proposed theory

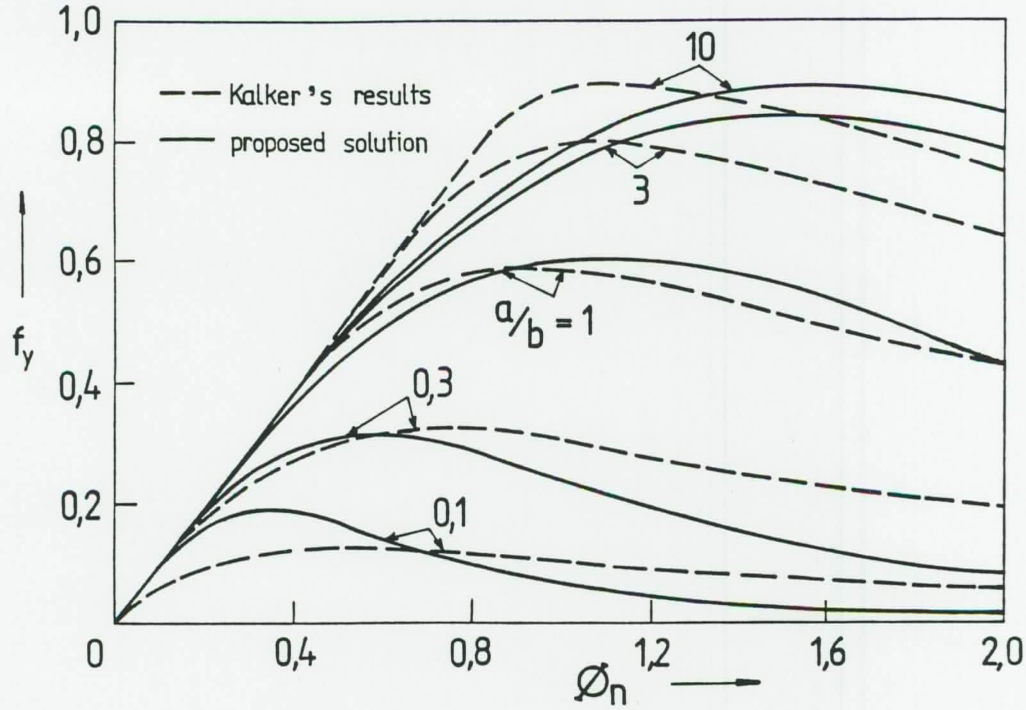


Fig. 3.
Comparison of non-dimensional lateral force caused by pure spin according to Kalker and according to the proposed theory

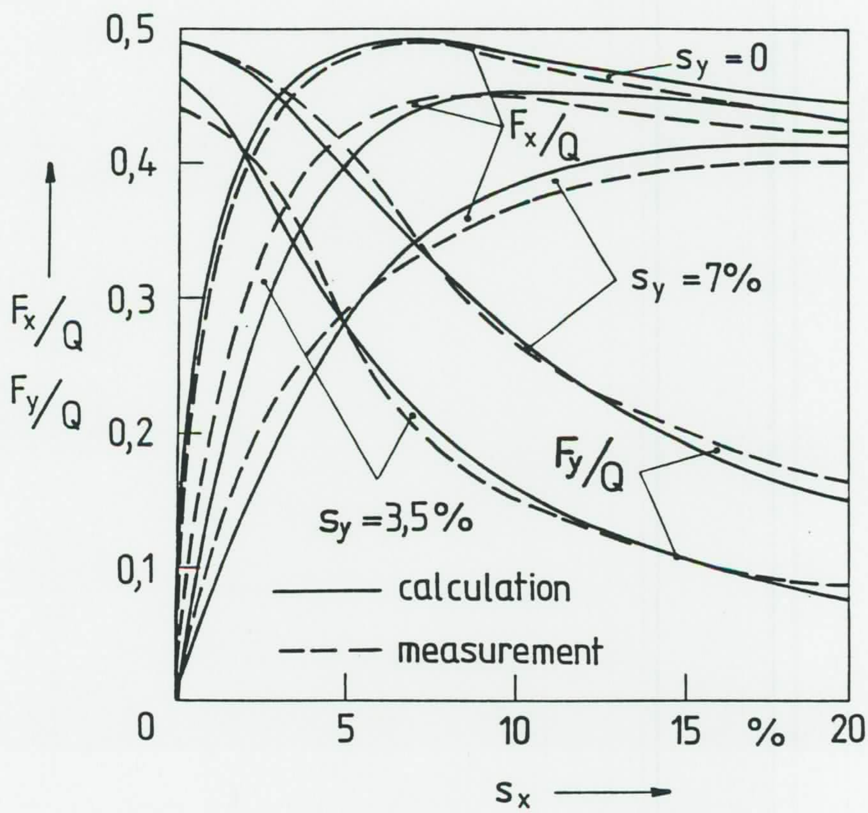


Fig. 4.
Comparison of calculation and experimental results measured on the model roller rig